Jillian and the Stopwatch

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It was a big moment for Jillian! After months of diligent work, she had mastered her Addition Facts—putting her at the top of her group. Her teacher, Mrs. Easterly was overjoyed at this most joyous moment for Jillian. Jillian was a little reluctant to show her smile, but Mrs. Easterly knew from that slowingly emerging smile, that Jillian felt a great pride bursting forth. Dr. Miller, known as the Math Magician, a bit more reserved than Jillian, did a superb job in concealing his ballooning delight, as he had seen how Mrs. Easterly and Jillian had worked with each other to reach this pinnacle of success for both of them.

But now, Jillian had entered the group of super math students led by Jacob and Charles who had achieved this goal shortly before her and were now working or mastering their multiplication facts. How good were they? Well, Mrs. Easterly was planning for her second grade students to challenge the third grade students in another classroom.

Here was Jillian moving into this very competive group of six boys. Mrs. Easterly had already helped develop her a strong confidence in herself. Could Jillian meet the challenge? In her first attempt at that tasked, she took a special test designed by the Math Magician. She got 46 of the 48 questions correct—missing only two. She took about five seconds per question. The Math Magician had set the first of three goals which was to do each multipliction in 3 seconds each.

Jillian needed a strategy. She knew that zero times any number resulted in zero as the answer, and that one times any number resulted in the number as the answer. Wait a minute! She also knew that multiplying by two was the same as adding a number to itself, and that she had already mastered addition. Mrs. Easterly had shown her how to practice a set of math facts. This was the little mini quiz that Jillian gave herself:

$$\begin{array}{cccccccc} 9 & 2 & 8 & 3 & 7 & 4 & 6 & 5 \\ \underline{\times 2} & \underline{\times 2} & \underline{\times 2} & \underline{\times 2} & \underline{\times 2} & \underline{\times 2} & \underline{\times 2} & \underline{\times 2} \end{array}$$

On her first try, it took her 32 seconds. Jillian was a little puzzled, because she knew that if those were addition facts, she could had done it in 12 seconds. Mrs. Easterly had taught her how to study this. So, Giulianna rewote the test as this:

$$\begin{array}{cccccccc} 9 & 2 & 8 & 3 & 7 & 4 & 6 & 5 \\ \underline{\times 9} & \underline{\times 2} & \underline{\times 8} & \underline{\times 3} & \underline{\times 7} & \underline{\times 4} & \underline{\times 6} & \underline{\times 5} \end{array}$$

Sure enough, on the second attempt she did it in 11 seconds.

So, she wrote the first quiz as two:

$$9 \quad 2 \quad 8 \quad 3 \qquad\qquad 7 \quad 4 \quad 6 \quad 5$$
$$\underline{x2} \; \underline{x2} \; \underline{x2} \; \underline{x2} \qquad\qquad \underline{x2} \; \underline{x2} \; \underline{x2} \; \underline{x2}$$

She tried the first one, and on the first go she did it in 6 seconds. The same for the second—6 seconds. Now she did them both together in 11 seconds.

Jillian thought about this for a while. It was probably this notation for multiplication that had initially stumped her. She was ready for the next challenge. Mrs. Easterly let the Math Magician show Jillian a special math trick. He asked Jillian to practice this set of examples of numbers summing to 10:

$$9 \quad 8 \quad 7 \quad 6 \quad 5$$
$$\underline{+1} \; \underline{+2} \; \underline{+3} \; \underline{+4} \; \underline{+5}$$

She learned the 9+1, 8+2, 5+5, quickly but needed a little practice to learn the 7+3, and the 6+4. Then the Math Magician said 9 and she said 1, he said 4 and she said 6. Jillian liked this trick. Dr. Miller then taught Mrs. Halliday, a kindergarten teacher how to teach this to her class. So now, Jillian was given this row of numbers to which she had to do the same trick with each number:

$$9 \quad 2 \quad 8 \quad 3 \quad 7 \quad 4 \quad 6 \quad 5$$

1   8   2   7   3   6   4   5   she wrote this

"That was fun", she thought to herself. Now in front of each of these numbers she had to write in front of them (concatenate) one less then the number:

$$9 \qquad 2 \quad 8 \quad\; 3 \quad\; 7 \quad\; 4 \quad\; 6 \quad\; 5$$

81   **1**8   **7**2   **2**7   **6**3   36   **5**4   **4**5

Then the Math Magician gave Jillian this set of problems:

$$\begin{array}{cccccccc} 9 & 2 & 8 & 3 & 7 & 4 & 6 & 5 \\ \underline{\times 9} & \underline{\times 9} & \underline{\times 9} & \underline{\times 9} & \underline{\times 9} & \underline{\times 9} & \underline{\times 9} & \underline{\times 9} \end{array}$$

Jillian said that she had not done this set of problems yet. Mrs Easterly told Guilanna that she has just done it a few minutes ago. Just put one less than the number down first and then put the tens complement down second as you did previously. Jillian did a she was instructed and her mouth dropped open for in just a few moments, she had started mastering her nines.

What do you think the Math Magician did next? Yes, he went back to Mrs. Halliday's class to show the Kindergaten students how to do the same thing. While he was about this, he also showed them how to modify this trick so that they could add nine to a number.

Mrs. Easterly gave Jillian the math quiz again, and this time Jillian did the quiz 52 seconds faster. Mrs. Easterly was a bit of a math magician too. She gave Jillian a stop watch as a present. Now Jillian could go home and give these quizzes to herself. She now had a way to measure herself.

She could now do her zeros, ones, twos, and nines all in 50 seconds—about 1 and one half seconds per fact.   Charles and Jacob are watching Jillian catching up to them like a runaway train on a straight track. They discovered that they could use their parent cell phone as a stop watch.

A number of years ago, a third grade student named Brian Keyes showed Dr. Miller how to multiply by four by doubling the number and then doubling the answer. Dr. Miller called this trick Double Trouble. He shared this with Mrs. Easterly who then shared it with Jillian. Nothing beats passing it forward. Now, Charles and Jacob were a little slow with the fours, so Jillian had the opportunity to catch up to them using her new stop watch.

Mrs. Easterly wrote the first line and Jillian wrote the next two lines:

9	2	8	3	7	4	6	5	
18	4	16	6	14	8	12	10	multiplying by 2
36	8	32	12	28	16	24	20	multiplying by 4

Jillian noticed that she had trouble with 4x9 and 4x8, the same as did Jacob and Charles. She had to learn to carry the one. So 18+18 was not 26, but 36 because

you had to carry the 1 from 9+9—adding it to the two making it a 3. The same was true of the eight. She then just practiced these two products until she had them memorized.   She also noted that she could use the 9 rule, thus using the commutative property-turn around facts.

Jillian was looking for herself to becom a math magician. Mrs. Easterly asked Guiliana to write down half of the following set of numbers:

2 4 6 8 18
1 2 3 4 9   this what Jillian wrote

Then she said do the same thing, but concatenate a zero after the number.

2   4   6   8   18
10   20   30   40   90   this what Jillian wrote

Then Mrs. Easterly wrote:

2   4   6   8   18
x5   x5   x5   x5   x5
10   20   30   40   90

Jillian looked in amazement as she said, "Multiplying even numbers by five is easy." Then Mrs. Easterly wrote:

2   4   6   8
x5   x5   x5   x5
10   20   30   40

2   4   6   8
x6   x6   x6   x6
12   24   36   48

Jillian looked closely and observed that multiply by six was like multiplying by five except you concatenate the number to the halved number instead of zero. Mrs Easterly then told her that when you multiply by 5 or 6 times an odd number, you drop down to the even number, follow the 5 or 6 rule and then add a five for multiplications by 5 and 6 for multiplications by 6. For example:

$$\begin{array}{cc} 3 & 2 \\ \underline{\times 5} & \underline{\times 5} \\ 15 & = 10+5 \end{array} \qquad \begin{array}{cc} 3 & 2 \\ \underline{\times 6} & \underline{\times 6} \\ 18 & = 12+6 \end{array}$$

Jillian looked over all that Mrs. Easterly taught her, and realized that she did not cover the threes. Mrs Easterly wrote the following:

$$\begin{array}{cccccccc} 9 & 2 & \textbf{8} & \textbf{3} & \textbf{7} & 4 & 6 & 5 \\ \underline{\times 3} & \underline{\times 3} & \underline{\times 3} & \underline{\times 3} & \underline{\times 3} & \underline{\times 3} & \underline{\times 3} & \underline{\times 3} \end{array}$$

Jilian asked her, if there were any tricks for the threes. Mrs. Easterly said that if she used her turn around facts, that the only ones that she did not kmow where 3x8=24, 3x3=9, and 3x7=21. If you want you could just add 8, 3, or 7 to itself three times.

"By the way", Mrs. Easterly said, "I did not teach you the following:

    49 = 7x7   Dr. Millers favorite football team
    56 = 7x8   counting 5, 6 , 7, 8
    64 = 8x8   I 8 and 8 until I got sick on the floor
    42 = 6x7   6 & 7 rode to heaven on a bicycle buikt four two.
    16 = 4x4   You can drive your mom's 4 by 4 until you are sixteen.

Jillian then went on to learning her multiplication facts and to her delight started challenging Charles and Jacob. They learned that if they wanted to stay competitive, that they were going to practice and wished that they had gotten a stop watch too.

During this period of time, Dr. Miller was sharing his math experience and testing tools that he had developed, with Mrs. Easterly. These tools gave Mrs. Easterly a way of measuring the success of her students and herself. The goal is to have the students go from taking five to ten seconds to get an answer to 1 second. One could see that even 2 seconds per fact would also be good. With these tools, Mrs. Easterly could also measure how well each student was doing and seeing exactly where they needed help. It also allowed the students to measure themselves. Thus the tests became contests for the student to compete with him/herself. If

one thinks about it, if it takes 5 seconds per fact, it would take the student about four minutes to do the tests. If we assumed that if took 20 tests to master the facts, it would take 80 minutes at that speed. By teaching the students fast ways to get the answer, she helps them learn faster. Thus while finger counting is a nice way to check the answer, it would not be as good a way to teach the facts.

From experience, about 15% of the second grade students master their addition facts. With this technique other teachers have improved that to 25%. However, Mrs. Easterly was able to raise that to 65%.   Having observed how Mrs. Easterly works with her students and her skill and love of math, it would be feasible that she would raise that number to 75%. She expects to have 50% of the students master their multiplication facts. How else will they be able to challenge the third grade students.

Jillian may have loved that stop watch as a present, but she gave us all a present by sharing her experience.