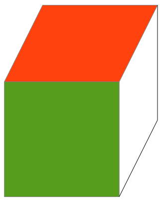
Pitfalls of Learning Mathematics

Illustrated by referencing the PBS video on NOVA: 'Zero to Infinity'



When looking at that cube, you most likely see the outside of a cube, where the corner of the orange , green, and white sides are near you. However some people are able to push that point into the paper so that you see the inside of a room where the orange side looks like the ceiling of the room. Some people have trouble with this optical illusion in a way analogous to people understanding mathematics.

New title for video: Not applying what we learned

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Pattern Recognition, Memorization, and how We think

Math has been taught in a way that makes it difficult to learn and understand. People are able to retain large amount of information by memorizing a selected portion and then recreate the rest by deductive reasoning. We cannot memorize all of possible ways of adding numbers, so we memorize a selected few and use an algorithm to derive the rest. Given a series of numbers such as 1 2 3, we can produce as many as we choose if we can see the pattern: 1 2 3 4 5 6 7   1 2 3 5 8 13 17 1 2 3 5 7 11 13. Thus, given a rule or algorithm we can determine that we have a counting sequence, a fibonacci series, or a seqential list of prime numbers.

Understanding why something works—the rule--, we can remember the rule more easily and then have a way of checking what we have done. Thus, when we add a negative, number, it is the same as adding the positive number. The defintion of a negative number is -3 + 3 = 0, Since subtraction is inverse addition, 3=0 - -3. The rule now becomes obvious. So, why are we not given a useful defintion of negative number.

Secondly, we teach misinformation. We define zero as being nothing and give cherry picked problems to support the speculation. 3X0 = 0 is nothing makes sense here, yet 3^0=1 does not make sense if we think of zeo as nothing. Is 1 plus 1 =2 or 1. If we have one apple and someone gives us another, we have two apples. But, if we talk about herds of cattle, we have one herd (just larger).

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Addition

Addition is a counting process, but it would take to much effort to add by counting.
However, there are a few simple obsrvations.

Adding
0 does not change the answer
1 gives us the next number for large numbers, depends upon naming convention
2 skip count
3 memorize 3 added to 4 6 7 and think of 3 as 5-2
4 memorize 4 added to 4 6 7 and think of 4 as 5-1
5 memorize 5 added to 5
6 think of 6 as 5+1 and use commutative and associative properties
7 think of 7 as 5+2 and use commutative and associative properties
8 add 10 and count back 2 (adding 10 for single digit, place concatenate 1 before number
9 add 10 and count back 1

In summary:
We practice counting forward and backward from 0 to 19
We practice skip counting forward and backward from 0 to 18 and 1 to 19.
We practice using the associative and commutative properties.
We memorize 1+9, 2+8, 3+7, 4+6, and 5+5

Multiplication

Multiplication is an addition process, but it would take to much effort to perform all those additions.

3X2=2+2+2   3x2=0+2+2+2

However, there are a few simple obsrvations.

Multiplying by:

0   always 0
1   same number
2   adding number to itself
3   memorize 3x3 3x7 3x8
4   multiply by 2 and then result by 2
5   even: half number concatenate to 0   odd: add 5 to previous even
6   even: half number concatenate number   odd: add 6 to previous even
7   memorize 7x6 7x7 7x8
8   memorize 8x8
9 subtract 1 from number for tens and number from 10 for units

1	2	3	4	5
9	8	7	6	5

Summary

0 and 1 obvious
2 and 4 double and doulble double
5 and 6 even half followed bu0 or number
9 subtract 1 from number for tens and number from 10 for units
Memorize 3x3 3x7 3x8 7x6 7x7 7x8 8x8

Rationaling that zero is nothing

Cherry Picking:

23.1 before: 023.1 after 023.10 value not changed

OOPS! Bigger 230.1 smaller 23.01

More cherry picking:

23 203 place holder huh?

213 1 is a place holder

Pattern recognition:

One way	Another way
3X5=5+5+5	3x5=0+5+5+5
2X5=5+5	2x5=0+5+5
1X5=5	1x5=0+5
0X5= what's here	0x5=0

Dropping +5 but on last step 5 Always dropping +5

In the first way we had to guess that 0x5=0 because of believing zero to be nothing, but we discovered **a new defintion for mutiplication**.

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A new definition for multiplication

Skip jumping illustration (manipulative)

2 3 4 5 6

3x2=2+2+2
Counts the +s

0 1 2 3 4 5 6

3x2=0+2+2+2
counts the +s

Zero is nothing mentality (incorrect manipulative)

0 1 2 3 4 5 6
3X2=2+2+2

We are counting the numbers

Perspection is different; could be the difference between understanding and not understanding math.

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A Pit fall

$5^3=5x5x5$	$5^3=1x5x5x5$
$5^2=5x5$	$5^2=1x5x5$
$5^1=5$	$5^1=1x5$
$5^0=$?	$5^0=1$ correct but we need a proof

Note that we have been using a backward count to utilized pattern recognition

Is zero the middle of the number line

Every positive number has a matching negative number. By symmetry zero must be the center of the number line.

Any number can be the center, because for every number to the left, there is a number to the right.

If we have to choose, then we will use the symmetry argument for selecting zero.

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A Pit fall

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Zero is the number before one

Using a new premise

How we make our numbers:

0 1 2 3 4

0+1 1+1 2+1 3+1 Add 1 to previous number

1=0+1

3=1+2 commutative property

3=(0+1)+2 transitive property

3=0+(1+2) associative property

3=0+3 transitive property adding 0

0+1=1

0=1-1 subtraction

0+1=1

(0+1)-1=1-1 subtraction

0+(1-1)=1-1 associative property

0+0=0 transitive property

0=0-0 subtraction

0=-0 notation

0+0=0

3x0=0+0+0

=(0+0)+0

=0+0

=0

0 is the only real number in which its positive value equals its negative value.

A justification for choosing 0 to be at the center of the number line.

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Zero and infinity

What is 1/0? There is no integer that is multiplied by zero that will give 1.

Discovery of fractions and calculus.

a+1/2 a+1/4 a+1/8 ….. getting closer to a

1 / ½ =2 1 / ¼ = 1/ 1/8=8 a=0

As the fraction gets smaller, its reciprocal gets larger.
A very large number is infinity.
Thus 1/0=∞
However, 0=-0 1/0 = 1/-0 = -∞ This is why division by 0 is undefined.
You want more proof. Consider graph for y=1/x (hyperbola asymptotes)

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Understanding infinity
Uncertainty principle

A bee flies between two trains that are traveling towards one other. How far does the bee fly.

We calculate the distance that the bee flies each time it touches the other train. The calculations are infinite if we use this approach. We do the calculation using d=r x t Where r is speed of bee and t is the time it takes the trains to meet

If we have a dropping object, we can measure the accuracy of the speed if the measuring distance is large. We can measure the distant more accurately if the distances use to measure the speed are close. This concept lead to the Heisenberg uncertainty principle.

If the trains starts off at 300 miles apart with one train at 60 mph and the other at 30 mph, the trans will meet in 2 hours The bee is traveling at 80 mph will be traveling for 2 hours, so she travels 160 miles. While the math take forever, the bee will not.

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The identity Elements

$a+0=a$ $\quad\quad$ $a \times 1=a$

$0 \times a=0$ $\quad\quad$ 1^a=1 for a = integer

$1/0=\pm\infty$ $\quad\quad$ 1^(1/2)=± 1

To simplify the concept of 1 and 0, we call them the identity elements; 0 for addition and 1 for multiplication.

However, as we studied zero, we got to understand 1. A number before 1 is $.\overline{9}$.
Therefore, we change our defintion: **Zero is the integer before 1.**

We also come on this dilemma.

0 1 2 3 4 5 6
0 1 2 3

The spacing for the two number lines is different. We come to the conclusion that the numbers above are ordinal numbers (labels) and that the spacings are cardinal numbers. On a clock an ordinal number is 4 o'clock and a cardinal number is 1 hr, 1 minute, etc. In look at this example, besides introducing the concept of dimensions, it suggest that 0 may be the first or starting number. The elevators in Europe start at 0 and the US, they start at 1.

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Ordinal Cardinal Numbers and Dimensions (1 is not 1)

Ordinal number is a position- 2 o'clock
Cardinal number for arithmetic operations 1 hr   1 o'clock + 2hrs=3 o'clock

Conversion
3600 seconds=1 hr   60 seconds=1minute
3661 pictures at one picture per 3 seconds base 60
3661+3 sec=(3600+60+1)xsecondsx3=1:01:01x3=3:03:03

Volume of 1 cubic ft

1 in=2.54cm 12in=1ft 1000 cm^3=1 liter 1 cm^3=1 gram
1000gm=1 kilogram   1 kg=2.20462 lbs

(1 ft)^3=(12 in)^3=(12x2.54cm)^3=(30.48cm)^3=28316.8cm^3=28317grams
=28300x kg/1000=28.3 kg=28.3x2.205 lbs=62.4 lbs

1 liter=1000 gm=1 1kilogram 1 liter= 1.05669 quarts 4 quarts=1 gal

(1 ft)^3=(28300)gm^3=28300 liters/1000=28.3 liters=28.3x1.057 quarts-=29.9quarts
=29.9xgal/4=7.48 gallons

60 mile/hr= 60 x5280 ft/3600sec= 5280/60 ft/sec=88 ft/sec
Notice how the dimensions are part of the calculations

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Discovering Negative Numbers

We have been hammering away at how universal is the definition of zero being the number before 1. Let see how powerful this concept can be,

B	A	0	1	2	3
	B+1	A+1	0+1	1+1	2+1

$$A+1=0 \qquad B+1=A$$
$$A=0-1 \qquad B+1+1=A+1$$
$$B+2 \quad =0$$
$$B= 0-2$$
$$A=-1 \qquad B=-2 \quad \text{notation}$$

$$A+1=0 \qquad B+2=0$$
$$-1+1=0 \qquad -2+2=0 \quad \text{Definition of a negative number}$$

With these definitions, we can prove the rules of Adding and subtracting Negative numbers.

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$A+1=0$	$B+1=A$	
$A=0-1$	$B+1+1=A+1$	
	$B+2 \quad =0$	
	$B= 0-2$	
$A=-1$	$B=-2$	notation
$A+1=0$	$B+2=0$	
$-1+1=0$	$-2+2=0$	Definition of a negative number

With these definitions, we can prove the rules of Adding and subtracting Negative numbers.

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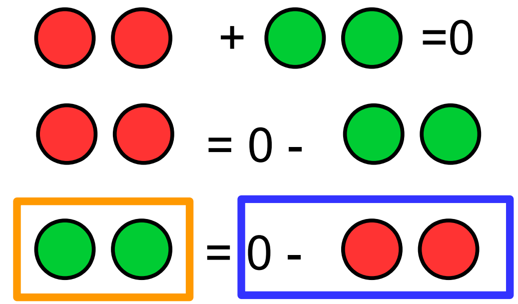
Rules for Negative Numbers

-2+2=0	defintion	-2+2=0
-2=0-2	subtraction	2=0- -2

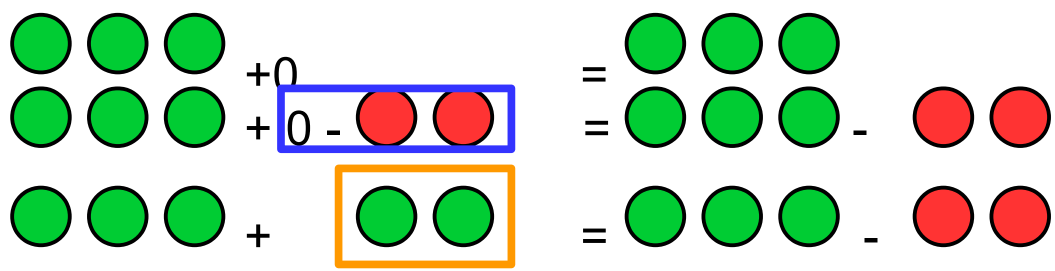


-2+2=0

-2=0 -2

2=0- -2

3+0=3	definition	3+0=3
3+0-2=3-2	subtraction	3+0- - 2 =3 - -2
3+(0-2)=3-2	association	3+(0- -2)=3- -2
3+-2=3-2	transitive	3+ 2 = 3 - -2



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More Rules for Negative Numbers

-2+2=0		-2+2=0	
-2=0-2		2=0- -2	

-3 +3 +-2+2=0+0=0	definition	-3 +3 +-2+2=0+0=0
-3 +-2+ 3+2=0	commutative	2 + -3 + 3 +-2=0
(-3+-2)+(3+2)=0	associative	(2+-3)+ (3+-2)=0
-3+-2=0-(3+2)	subraction	2+-3=0 -(3 + -2)
-3+-2=-(3+2)	transitive	2-3=-(3-2)

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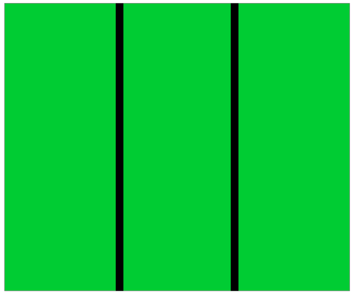
Discovering Fractions

1=1/3+1/3+1/3=3x1/3

Defintion 3x1/3=1
1/3 = 1 / 3 space2 to indicate / is a divison sign
3= 1 / 1/3 inverse of fraction

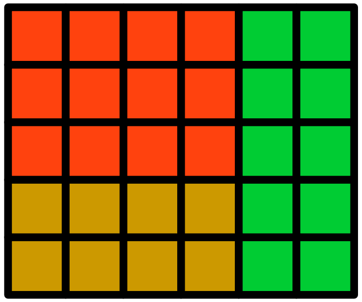
1/3+1/3=2/3



3x1/3=1 definition of division
2x3x1/3=2 multiplication
3x(2x1/3)=2 commutative/associative
2x1/3=2 / 3 division
2X1/3=2/3 notation

Multiplying fractions
3X 1/3 x 2x/1/2=1x1=1 definition
3X2 x 1/3 x 1/2 =1 commutation
(3X2) x (1/3 x1/2)=1 association
1/3 x1/2 =1 / 3x2 division
1/3 x 1/ 2 =1/ (3x2) transitive

3/5x2/3
=6/15



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Using Fractions

Adding fractions:

Distributive prop

$1/7+2/7+3/7=1\times1/7+2\times1/7+3\times1/7=(1+2+3)\times1/7=6\times1/7=6/7$

Multiply fractions:

$2/3\times5/7=(2\times5)/(3\times5)=10/35$

Adding unlike denominators

$2/3+5/7=1\times2/3+1\times5/7=7/7\times2/3+3/3\times5/7=7\times2/21+3\times5/21=14+15/21=29/21$

Mixed and Improper fractions

$2\ 3/7=2/1+3/7=7/7\times2/1+3/7=(7\times2+3)/7=17/7$

$$\begin{array}{l} 3+2/7 \\ \underline{\times\ 7+3/7} \\ 7\times3+7\times2/7 \\ \underline{\qquad 3\times3/7+2/7\times3/7} \\ 21+2+9/7+6/49=23+1+2/7+6/49 \\ =24+20/49 \end{array}$$

$23/7\times52/7$

$$\begin{array}{r} 52 \\ \underline{\times23} \\ 156 \\ \underline{104\ } \\ 1196 \end{array}$$

$$\begin{array}{r} \underline{\quad 24}\ R\ 20 \\ 49)1196 \\ \underline{98\ } \\ 216 \\ \underline{196} \\ 20 \end{array}$$

Most of work done mentally

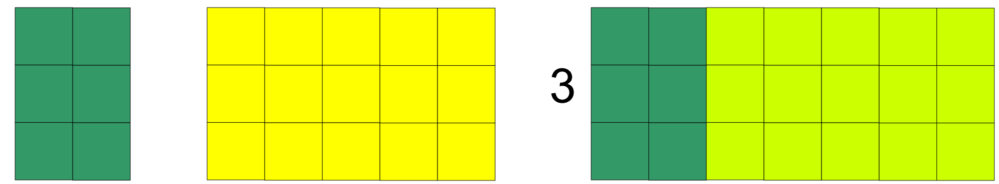
Very Long Multiplication

Powers of 10: 500x3000=5x100x3x1000=(5x3)x(100x1000)=15x100,000=1,500,000
Just count up the zero

Distributive Property 3x(2+5)=3x2+3x5
3X7 =6 +15

21= 21

2 5 7



3 3

3X2 + 3x5=3x(2 + 5) 3x7

```
 11      11
235     235      12347      12347      12347     12347
235     x 3      x     5    x   30     x 700     x    735   =700+30+5
235     705      61735     370410    8642900      61735
705                                              370410
                                                8642900
                                                9075045
```

```
     12347                        making use of previous work
     12735 =12000+735
   9075045
   24694
  12347
 157239045
```

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```
  12345
 x12345
  61725
 49380
37035
24690
12345
152399025
```

```
 123450+6
x123450+6
15239902500
    740700
    740700
        36
15241383936
```

123456x(10^12+10^6+ 1)

```
                   123456123456123456
                  x123456123456123456
                          15241383936    123456x123456
                    15241383936          123456x123456x10^6
              15241383936                123456x123456x10^12
              15241399177399177383936    123456x123456123456123456
        15241399177399177383936          123456x10^6x123456123456123456
  15241399177399177383936                123456x10^12x123456123456123456
  15241414418813596182290783113383936
```

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Number Structure

Base 10: 0 1 2 3 4 5 6 7 8 9

235.34=200+30+5+3/10+4/100
=2x100+3x10+5x1+3 / 10 + 4 /100
=2x10^+3x10^1+5x10^0 +3x10^-1+4x10^-2

Base 4: 0 1 2 3

231.3=2x4^2 + 3x4^1+1x4^0+ 3 x4^-1
=32+12+1+.75=45.75

231.3	45.75	2x 1024= 2048
x231.3	x45.75	2x 16= 32
20211	22875	3x 4= 12
2313	32025	1x 1= 1
20211	22875	2093
11302	18300	.01=1/16=.0625
200331.01	2093.0625	

Area of Similar Triangles

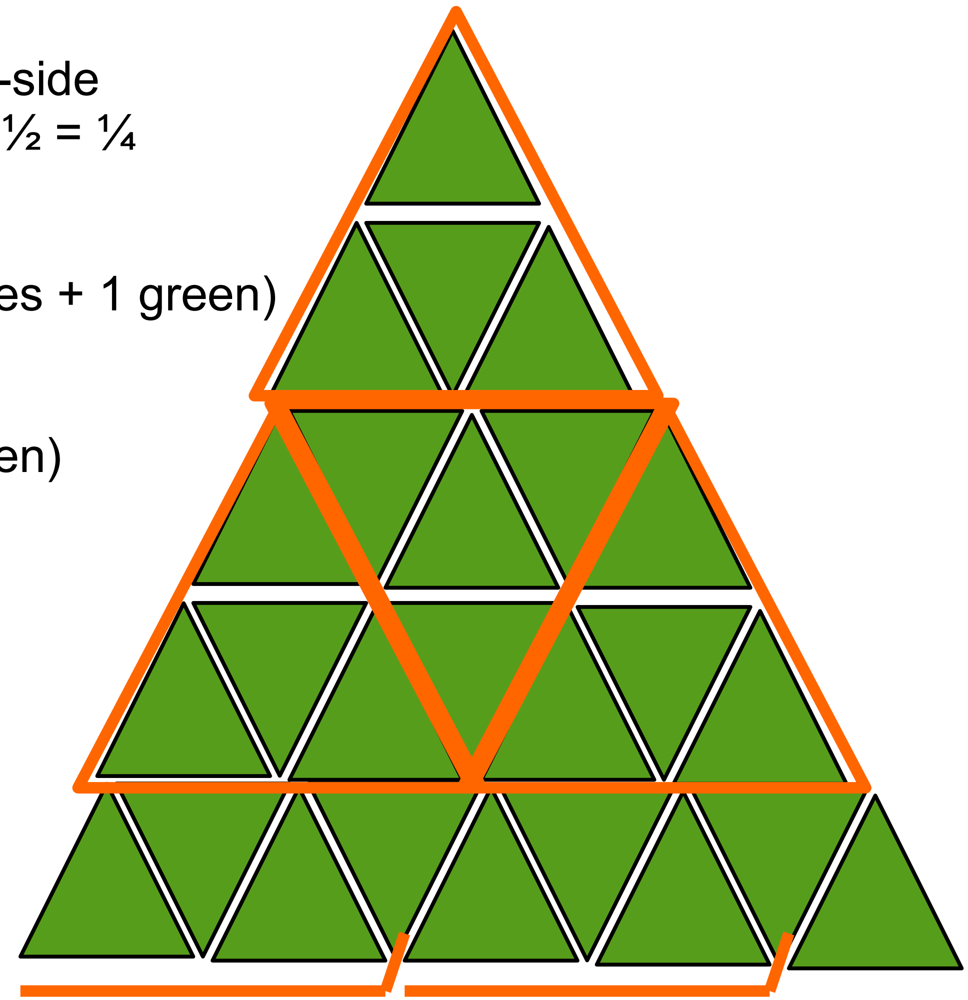
Each green triangle half of orange side-to-side
Area of green triangle proportional to ½ x ½ = ¼
4x ¼ = 1 = area of orange triangle

Area of last line = 9x1/4 =2 ¼ (two oranges + 1 green)

There are 4 orange triangles
Total area= 4 + 2 ¼ =6 ¼ = 25 / 4 (25 green)

2 1/2	2.5
X 2 1/2	x 2.5
2x2+2x1/2	125
2X1/2+1/2x1/2	50
4+1+1+1/4=6 1/4	6.25

21/2=5/2   5/2 x 5/2=25/4
There are 25 1/4 triangles



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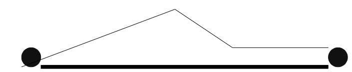
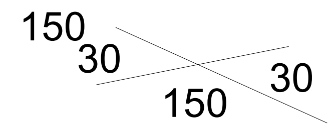
Geometry

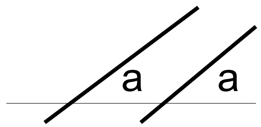
Straight Line
A straight line is the shortest distance between two points



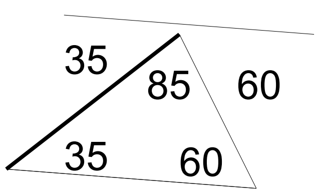
Intersecting lines
When two lines intersect the opposite angles are =.



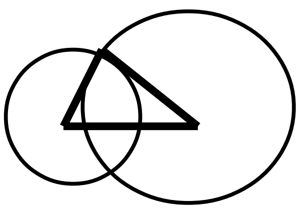
Parallel lines
Two parallel lines make the same angle with a third line



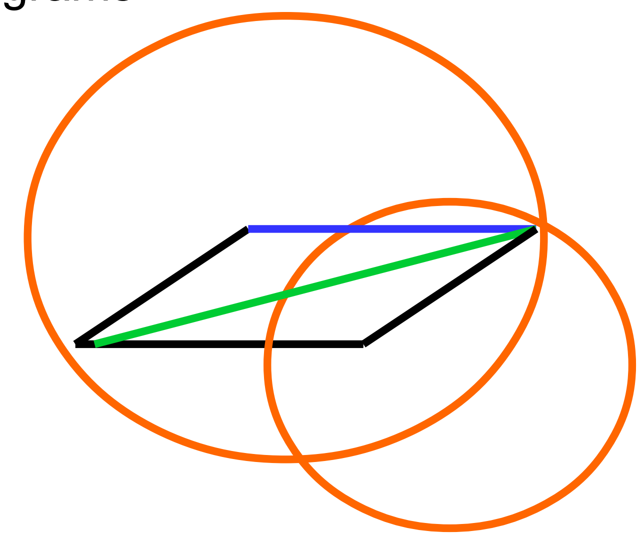
Sum of angles of a triangle
180 degree



Congruent triangles



Drawing parallelograms



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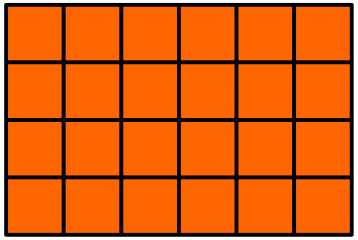
Area

The unit square (rt angles)

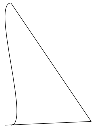


6x4

$Area=LW$ $L=6$ $W=4$ $Area=6x4=24$ sq units
$Perimeter=L+L+W+W=2(L+W)= 2(6+4)=20$ units

$L=s+a$ $W=s-a$ if $s=5$ $a=1$ $L=6$ $W=4$

$A=LW=(s+a)(s-a)=s^2-a^2$
$P=2(s+a+s-a)=2(2s)=4s$
A square is the biggest area for a given perimeter



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