Geometry Tutorial

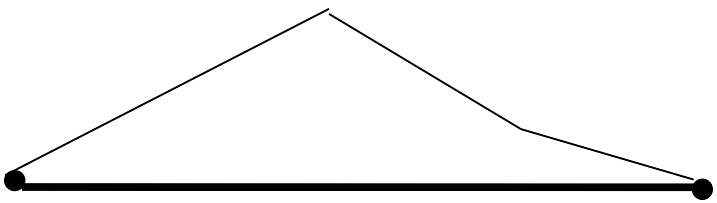
Internet and Classroom

Presentation

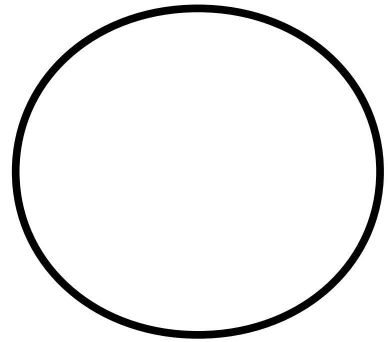
Geometry

Basics
Transformations
Lines & Angles
Triangles
    Congruent and Similar
Perpendicular and parallel lines
Quadralaterals
Pythagorean Theorem
Circles, chords and tangents
Trigonometry, St. lines & Graphs

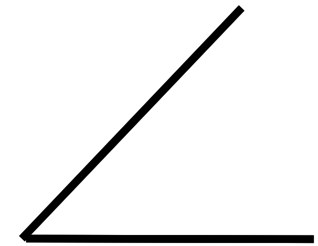
Basic Objects



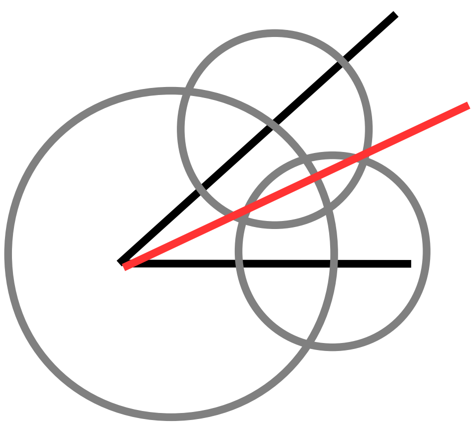
Straight Line



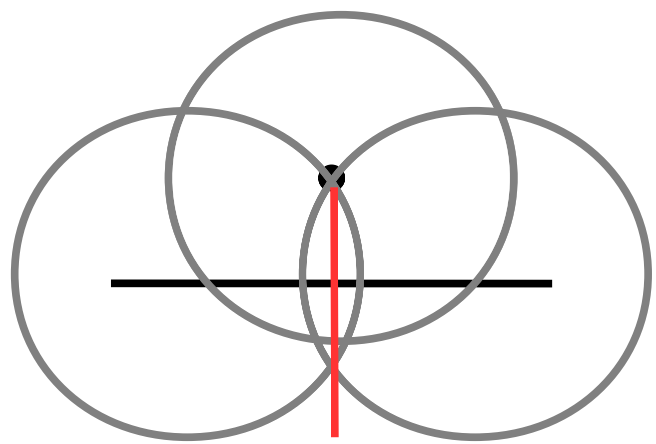
Circle



Angle



Bisect an angle



Drawing a perpendicular

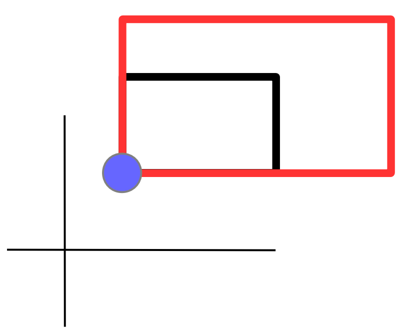
Transformations

Translation movement in x and y direction
Dialation change in size about one **point**
Rotation Movement around one point
Reflection **Perpendicular** distance from a line.

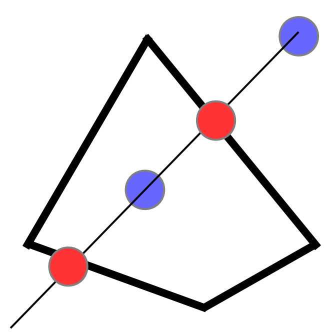
Objects are congruent with all except dialation
Objects are similar with dialation

Inside outside intersect with sides of polygon

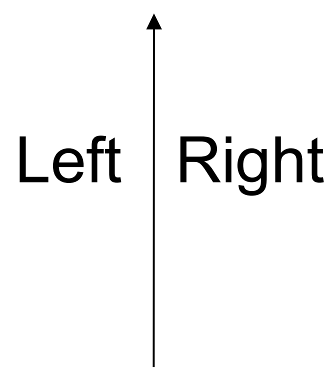
Right left of a line

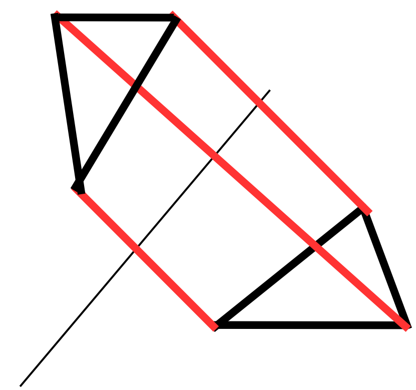


Magnification

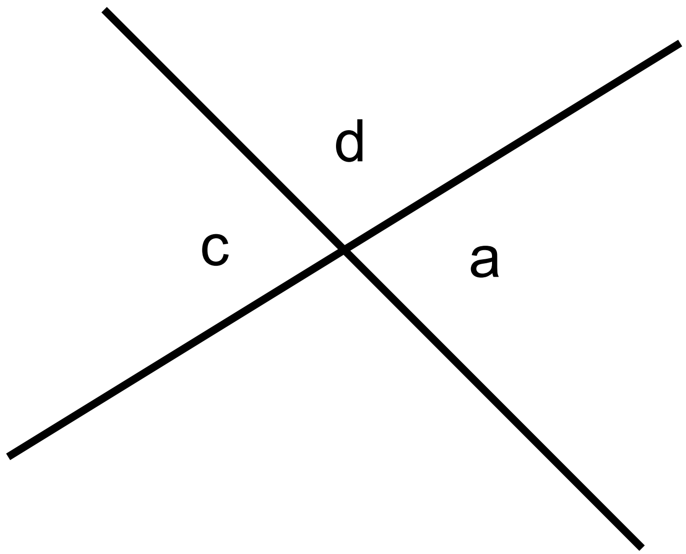


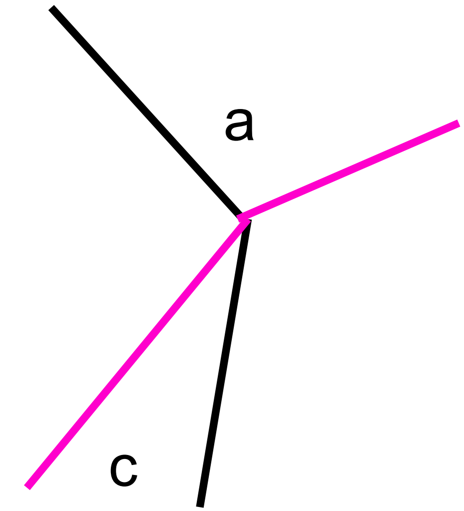
inside/outside





Reflection

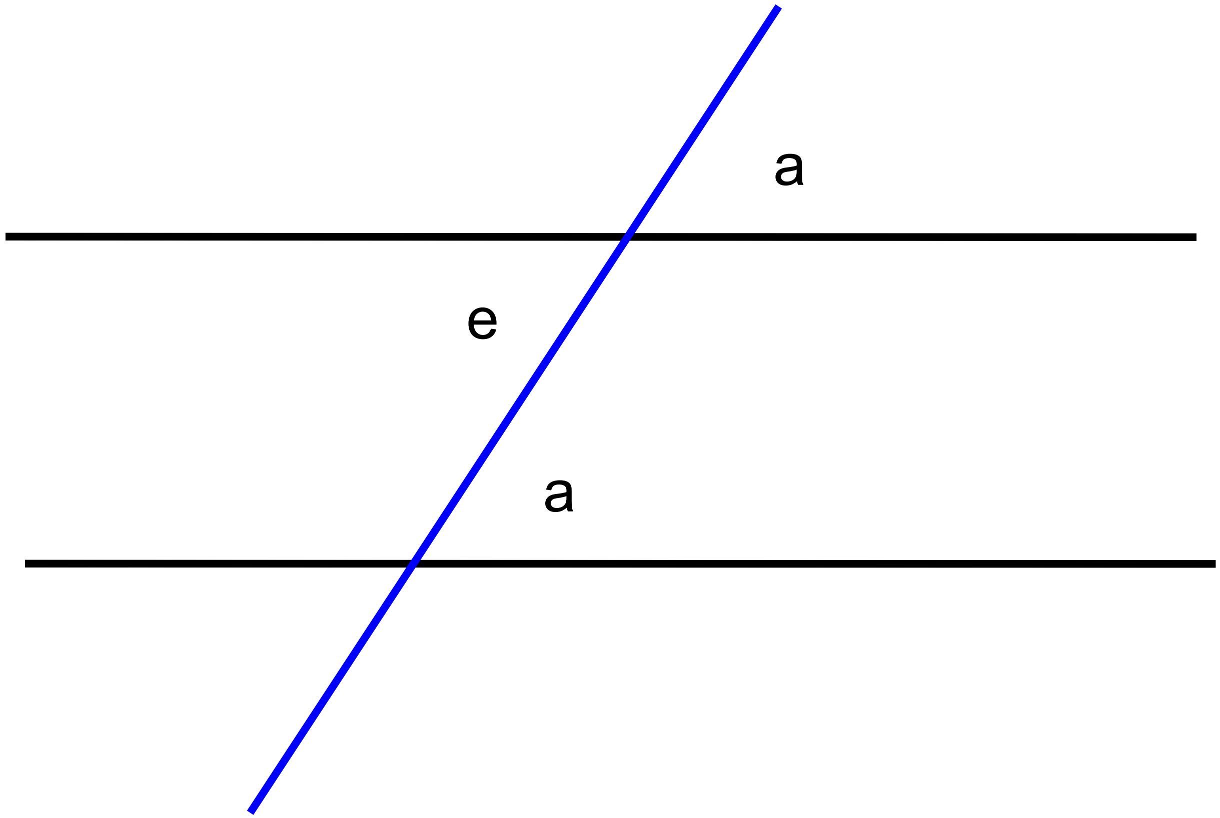




$a+d=180^0$ $a=180^0-d$

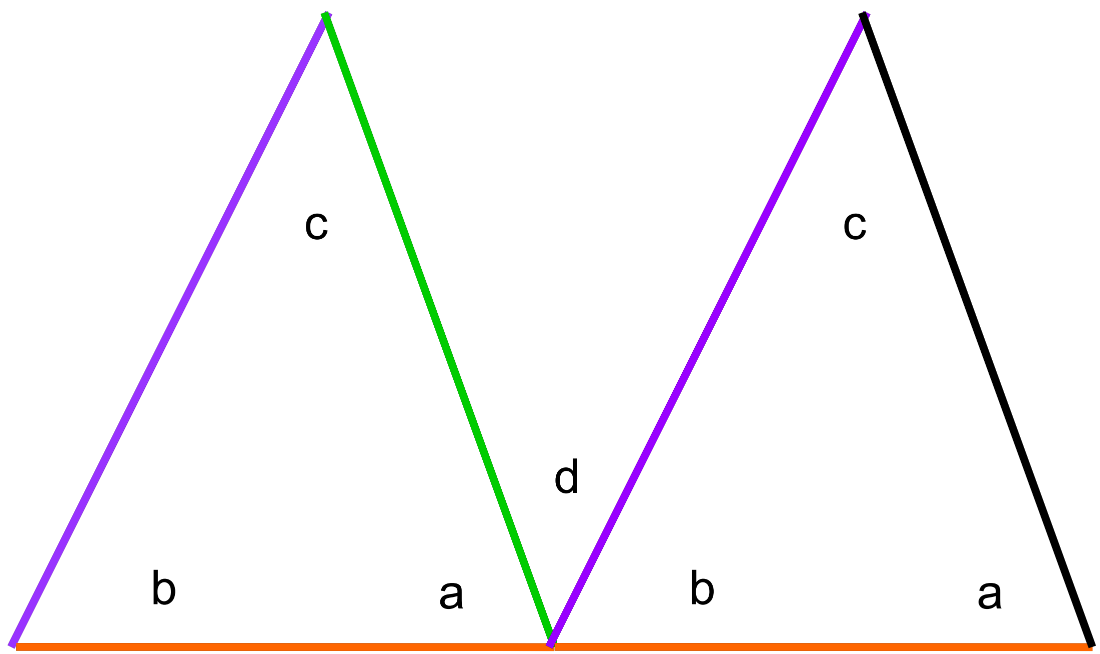
$c+d=180^0$ $c=180^0-d$

$a=c$



a=a	defintion of parallel lines
e=a	opposite interior angles of straight lines
	Alternate interior angles of parallel lines are equal

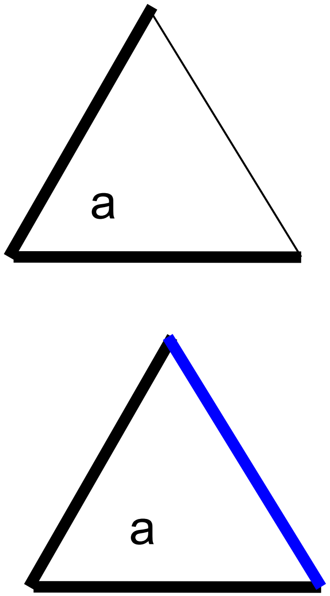
End of lesson1



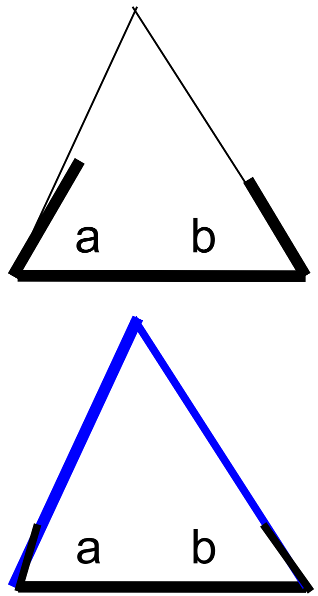
Purple lines parallel	angle b equal
Angles c=d	opposite interior angles of parallel lines
$a+b+d=180^0$	Straight line
$a+b+c=180^0$	Transitive property
b+d=b+c	Transitive property
	Exteriot angle egual sum of opposite interior angles

Congruent Triangles

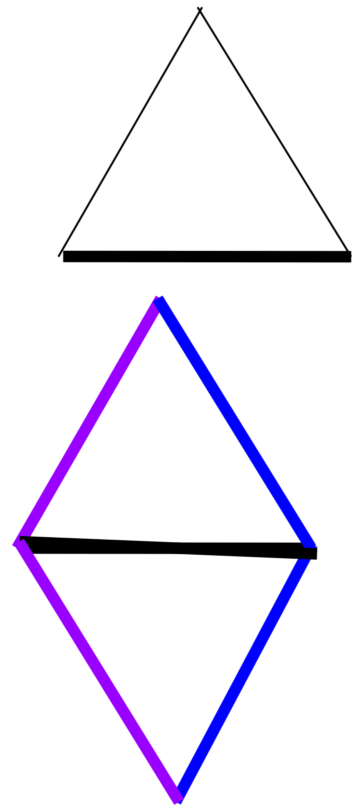
SAS



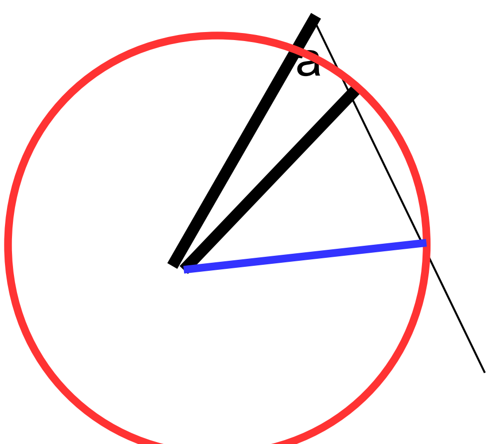
ASA

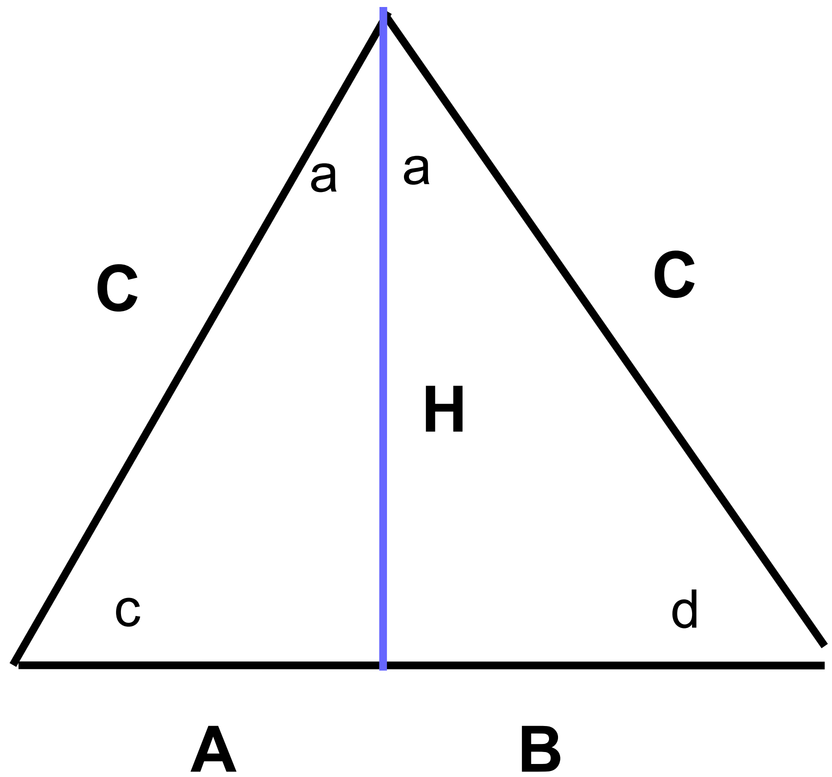


SSS

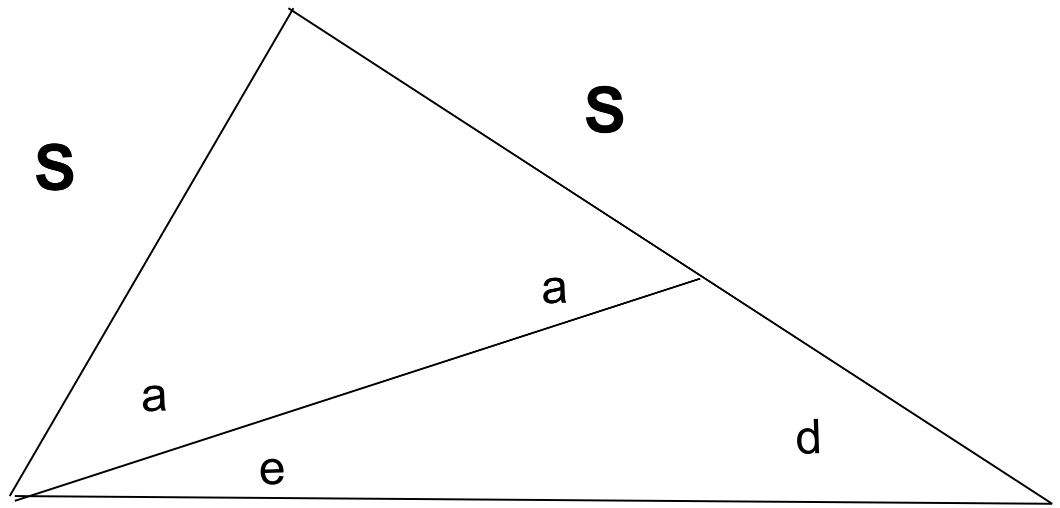


SSA not congruent



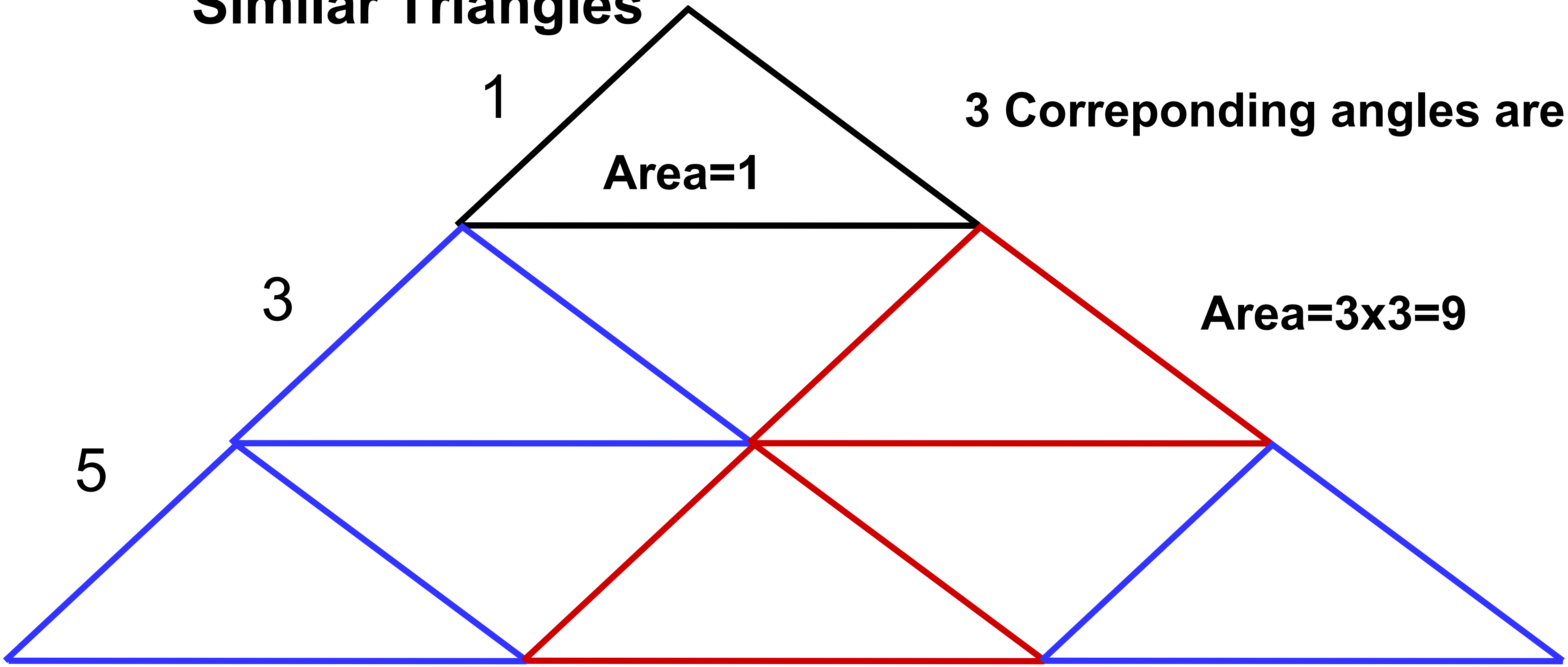


a=a	Bisect vertex angle
C=C	Given
H=H	Identity
SAS	Triangles congruent
A=B	Corresponding sides - Bisected base
c=d	Corresponding angles of congruent triangles



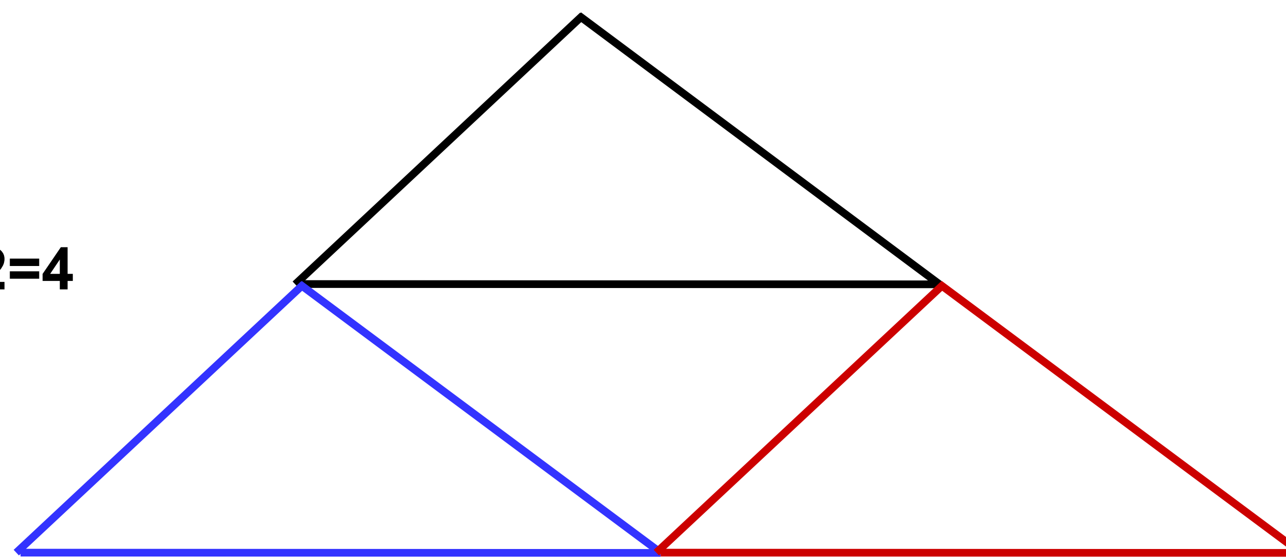
a=a	isosceles triangle
a=d+e	exterior angle of a triangle
d=a-e	
a+e>a-e	
a+e>d	opposite side across from larger angle

Similar Triangles



3 Correponding angles are =

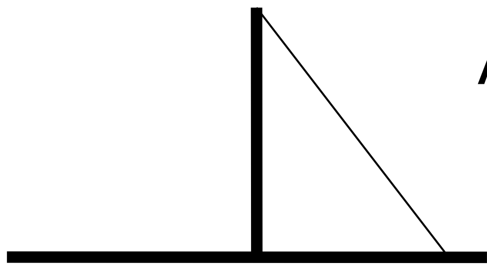
Area=2x2=4



AAA

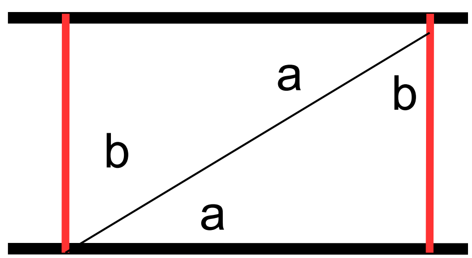
End of leson 2

Perpendicular and Parallel lines



A perpendicular is the shortest distance
between a point and a line.

The hypotenuse is opposite the
largest angle which is 90^0, so it's longer than the $\perp$.



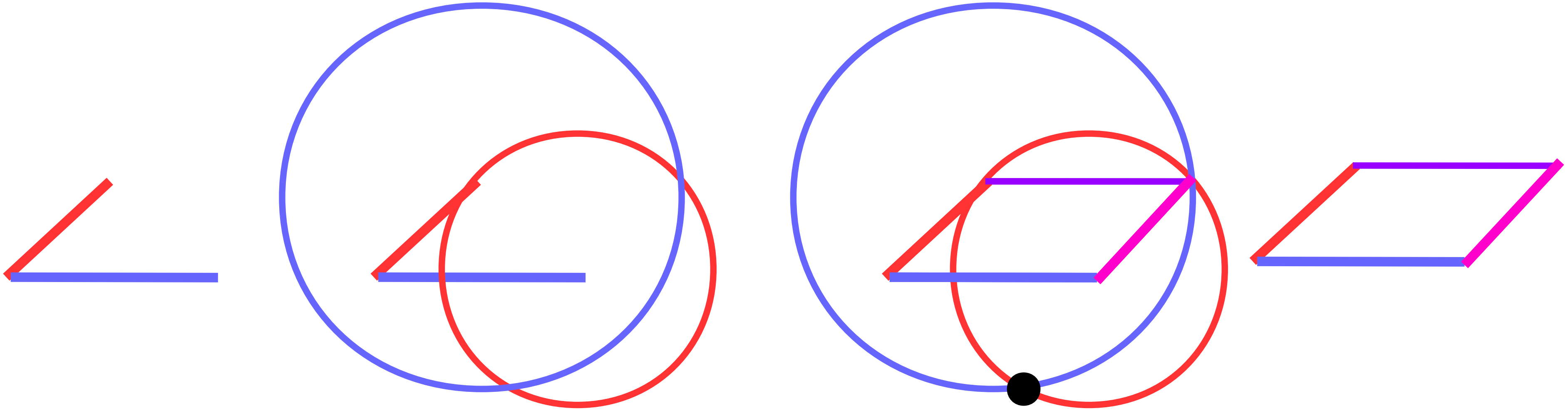
The a's are equal   interior angle of parallel lines
The b's are equal   perpendiculars are parallel
Triangles are congruent   ASA
Lengths of perpen. equal Thus lines never meet

Organizing our Quadrilaterals

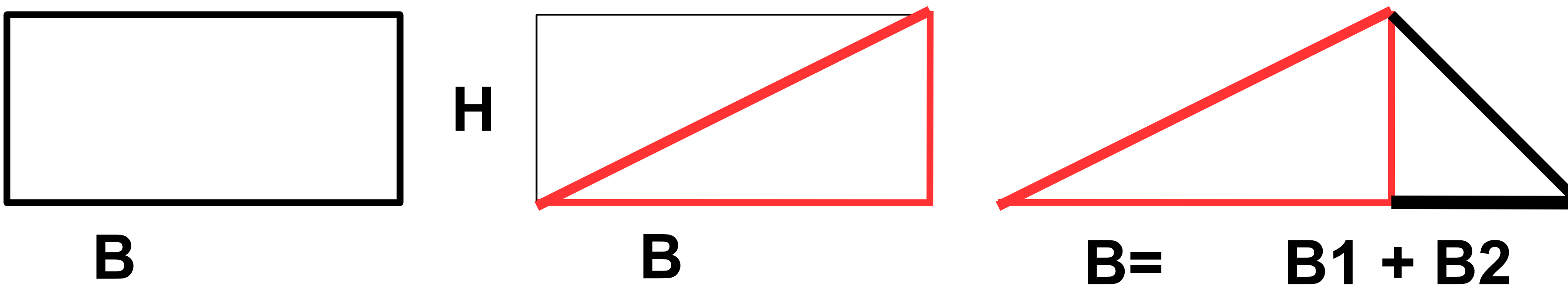
A kite and dart are four sided shapes in which the the adjacent sides are equal and the opposite adjacent sides are equal.
A dart is a kite with one concave angle.

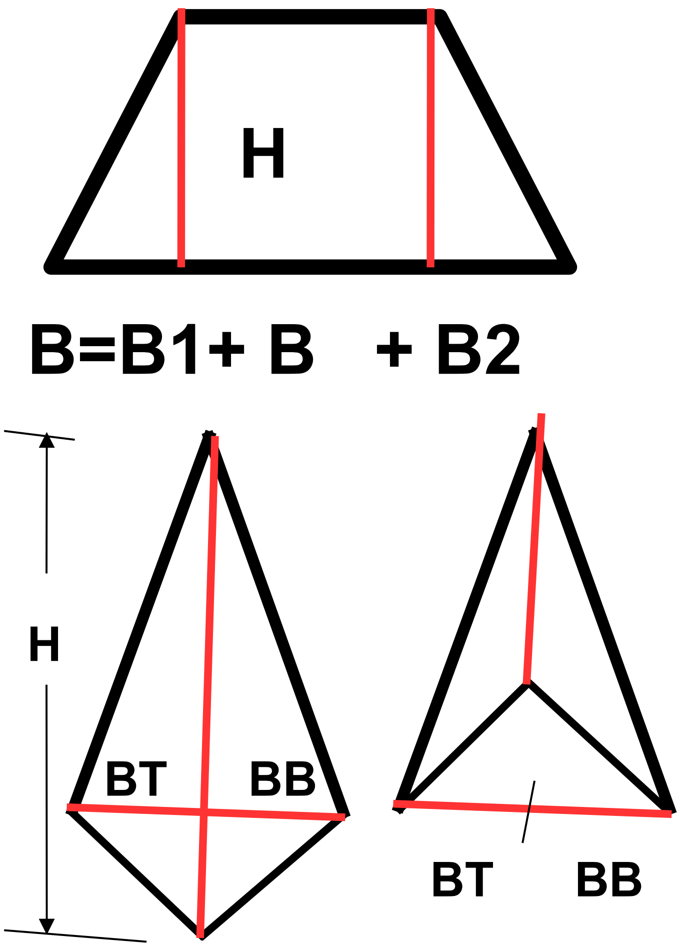
A trapezoid is a quadrilateral with one pair of opposite sides paralle

A parallelogram is a four sided shape in which the opposite sides are equal.
A rhombus is a parallelogram with equal adjacent sides. (A diamond is a rotated rhombus)
A rectangle is a parallelogram in which the angle of the two adjacent sizes is a right angle.
A square is a rectangle with equal adjacent sides.

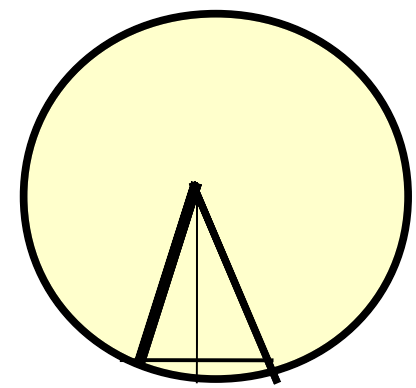


Area





Area=1/2 H*(BT + BB)
Trapezoid
Triangle BT=0
Rectangle BT=BB
Square BT=BB=H
Parallelogram BT=BB
Kite & Dart BT=BB



Area=1/2 RB*n
B*n=C=2πR
Area=πR^2

The Quadrilateral Family Tree
Summarization

Trapezoid

Parallelogram = opposite sides

Kite =adjacent sides

Rectangle right angle

Dart convex angle

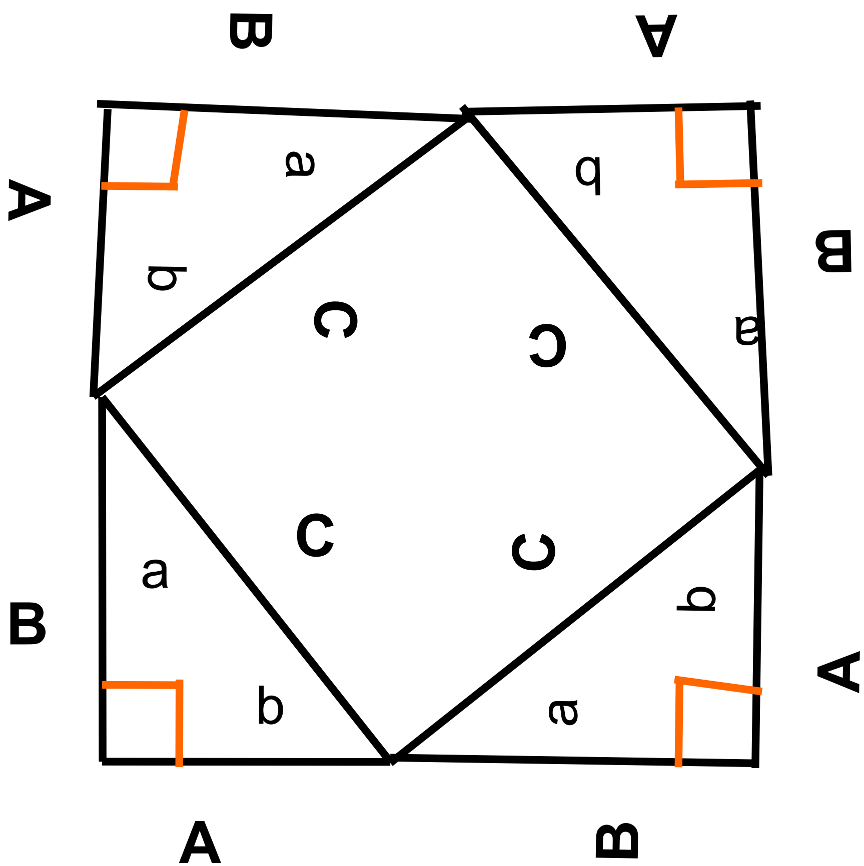
Square

Rhombus Equal sides

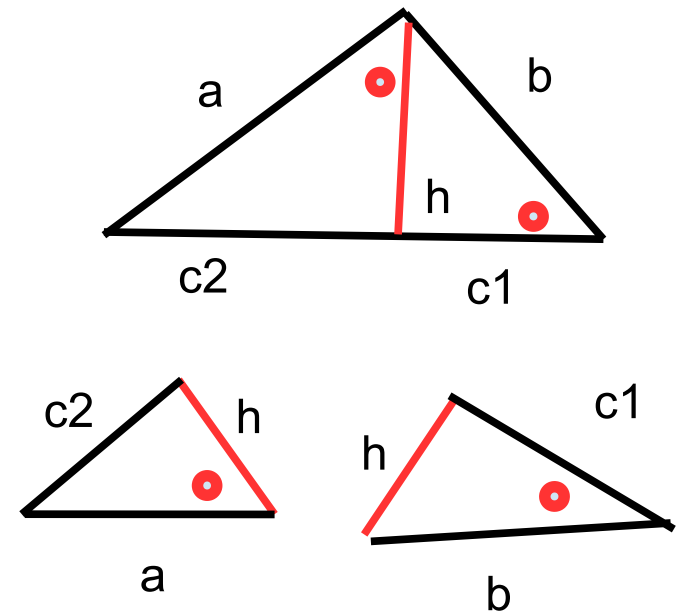
7 objects **Square** is a regular quadrilateral

End of leson 3

Pythagorean Theorem



$a+b+c=180$
$a+b=90$
$90+c=180$
$c=90$

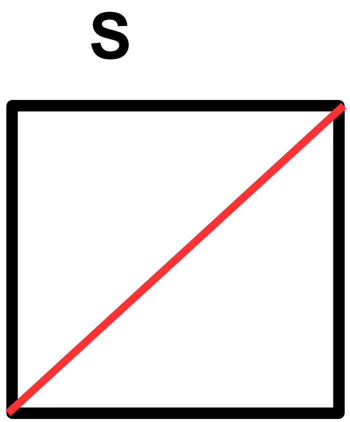


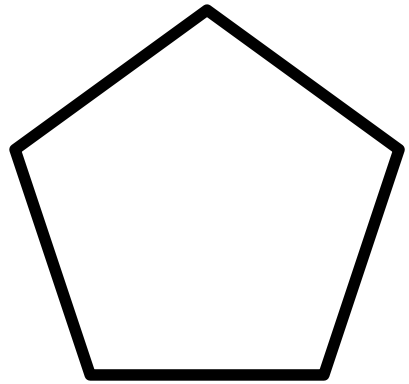
$c=c1+c2$
$c2/a=a/c \quad c1/b=b/c$
$c2/c=a^2 \quad c1xc=b^2$

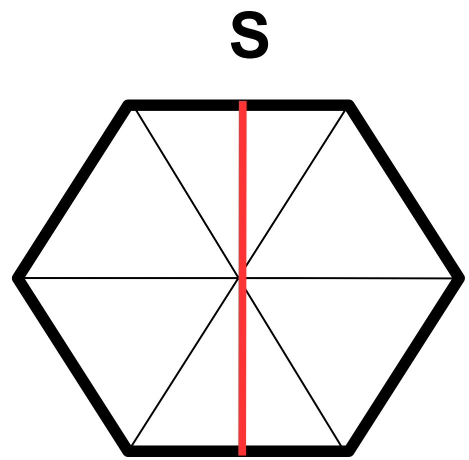
$cx(c2+c1)=a^2+b^2$
$c^2=a^2+b^2$

$(A+B)^2=C^2+4*\ 1/2(AB)$
$A^2+\cancel{2AB}+B^2=C^2+\cancel{2AB}$
$A^2 \qquad +B^2=C^2$

Ratio of Perimeter to Diagonal



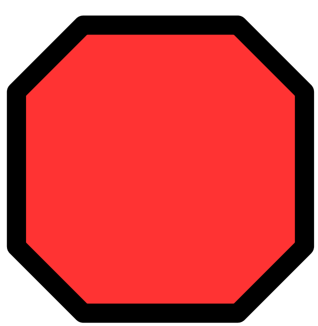




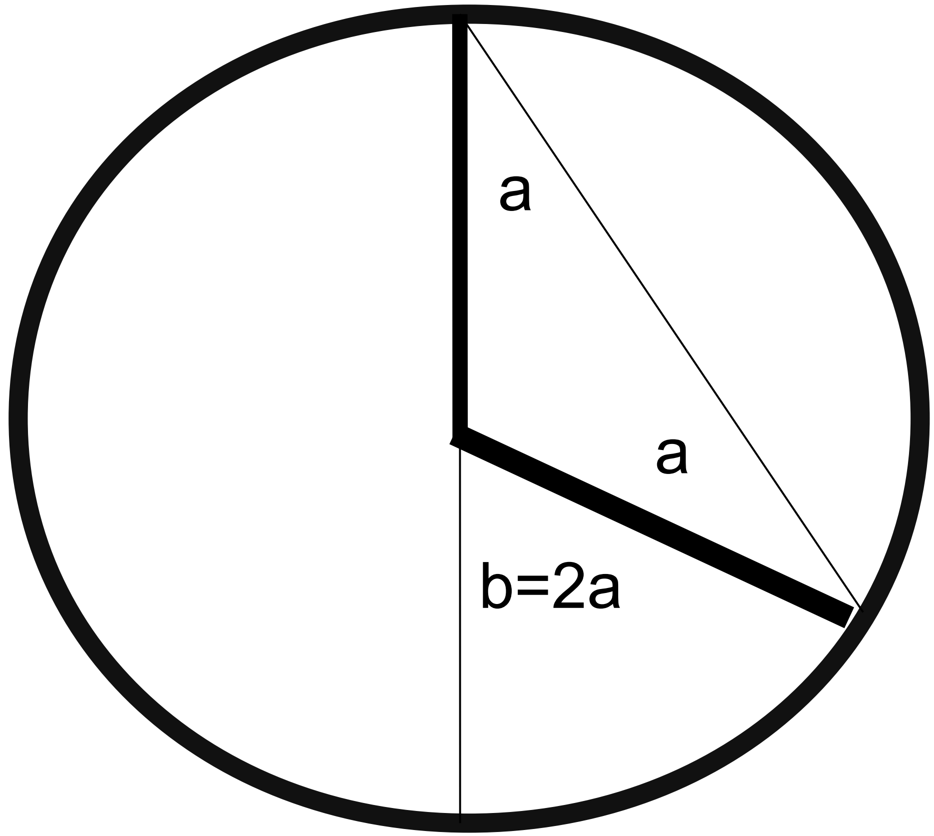
$D=2^{1/2}\ S$

$D=3^{1/2}\ S$

$C/D=4S/S=4$
$=4S/2^{1/2}\ S$
$=2\ 2^{1/2}=2.828..$



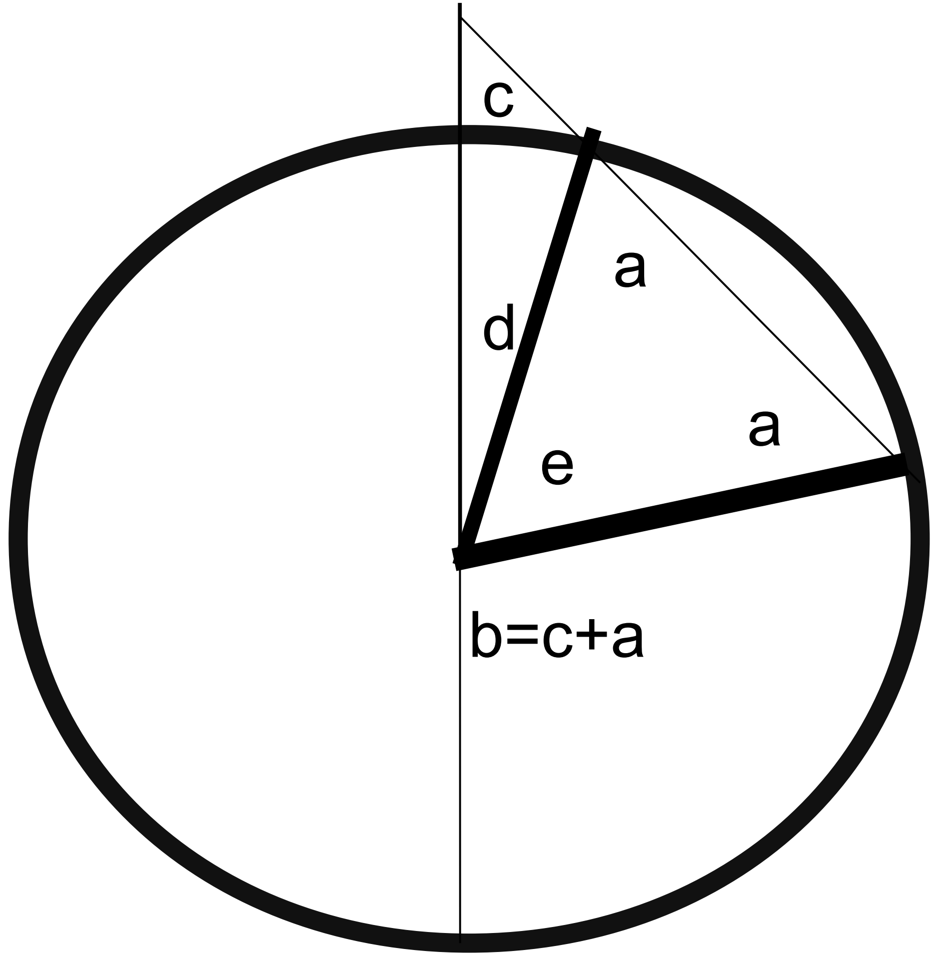
$C/D=6S/2S=3$
$=6S/3^{1/2}\ S$
$=2\ 3^{1/2}=3.46$



a=a base angles of isoceles triangle
b=2a Exterior angle of triangle = sum of opp. Interior angle
a=b/2 QED Inscribed angle of circle = ½ central angle
Described by circle.

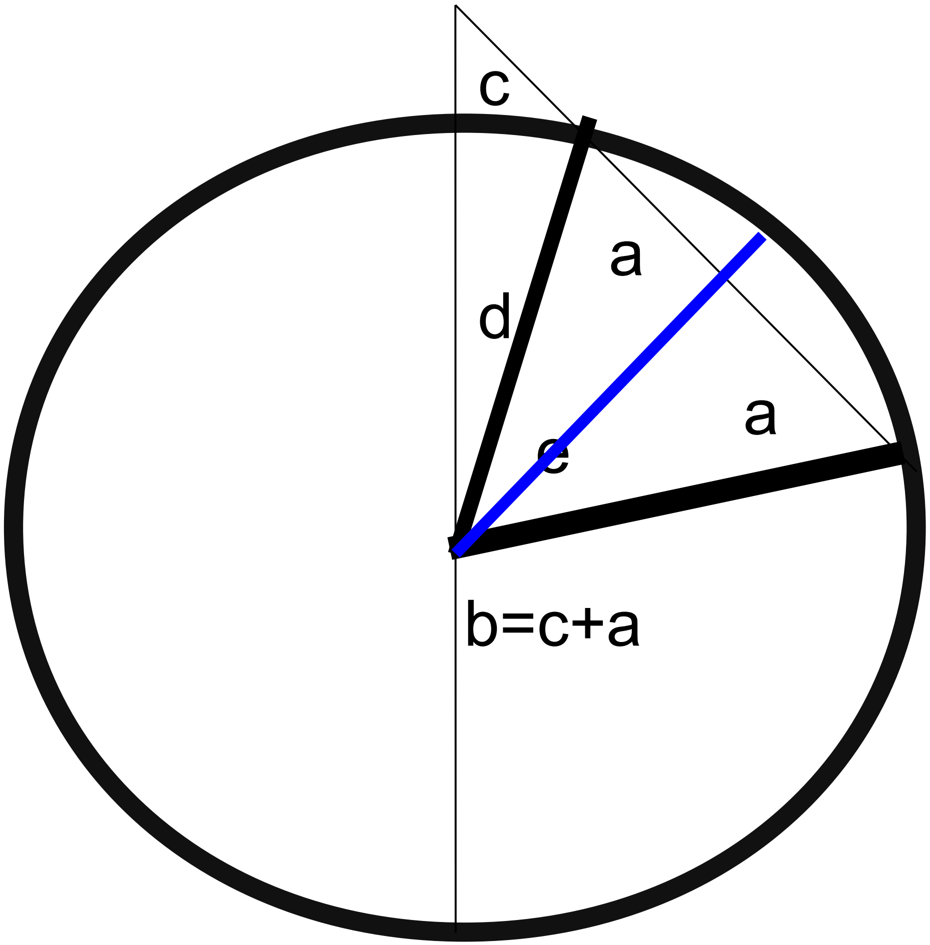
What is the measure of an angle inscribed in a semi-cirlce?

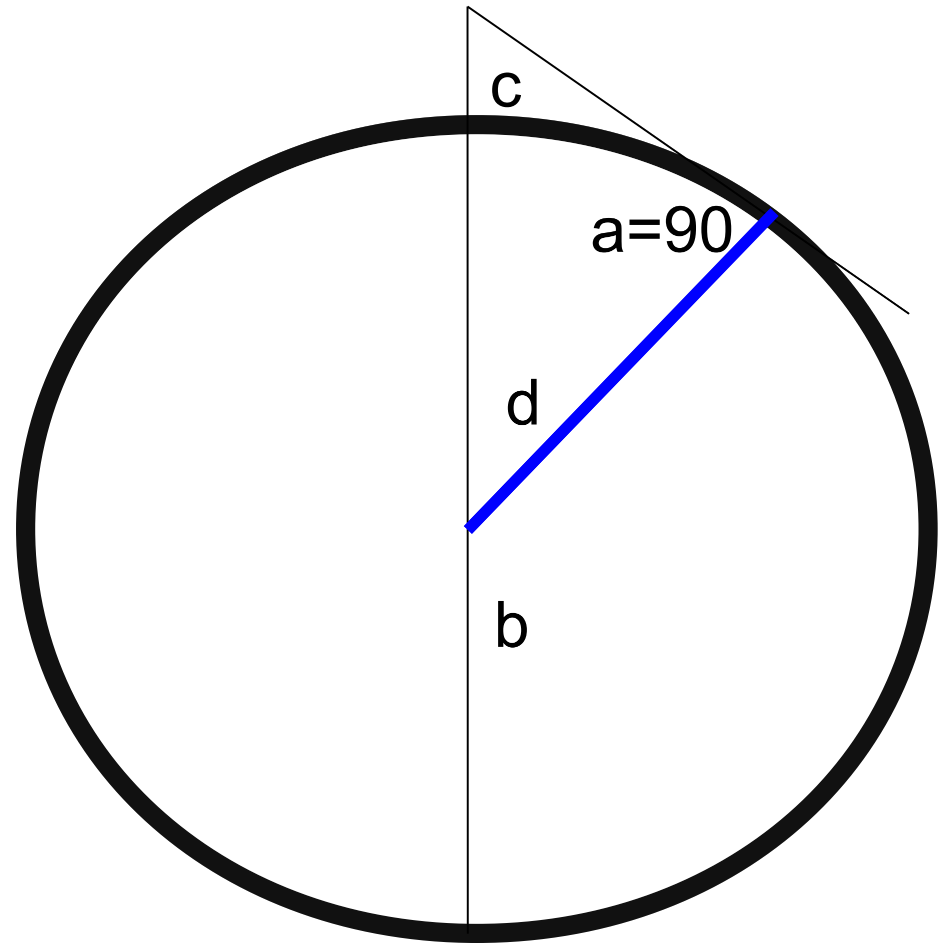
End of leson 4



a=c+d
b=c+a
b=c+c+d
c=(b-d)/2

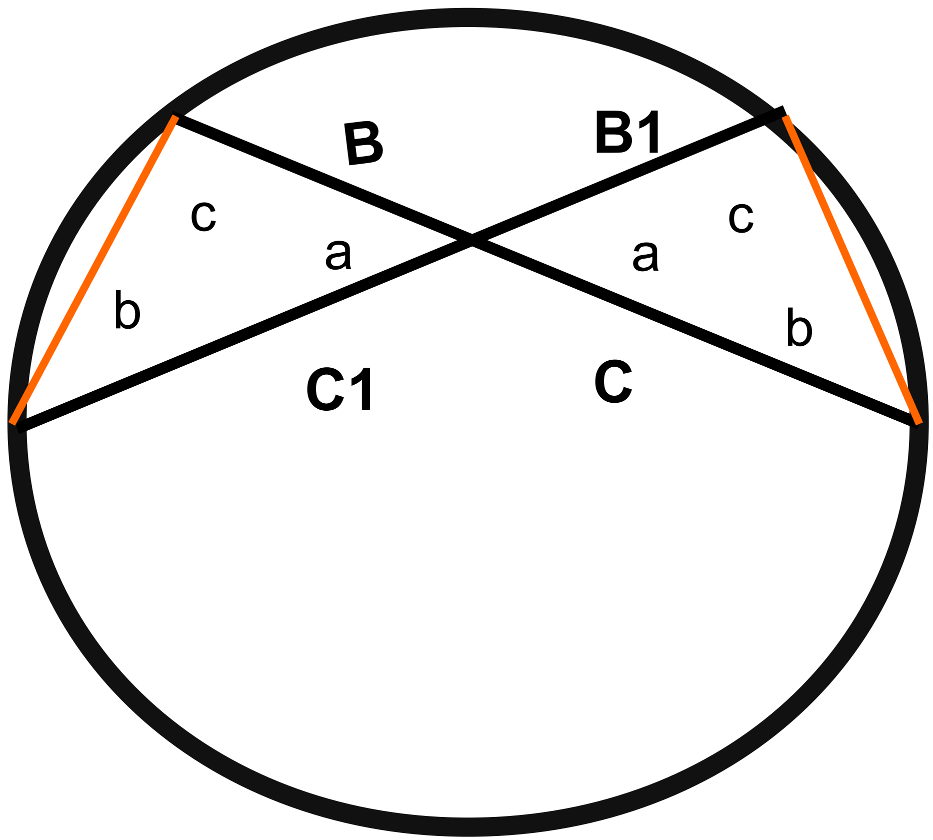
Note when c touches circle, d-0 and we validate proof for previous theorerm

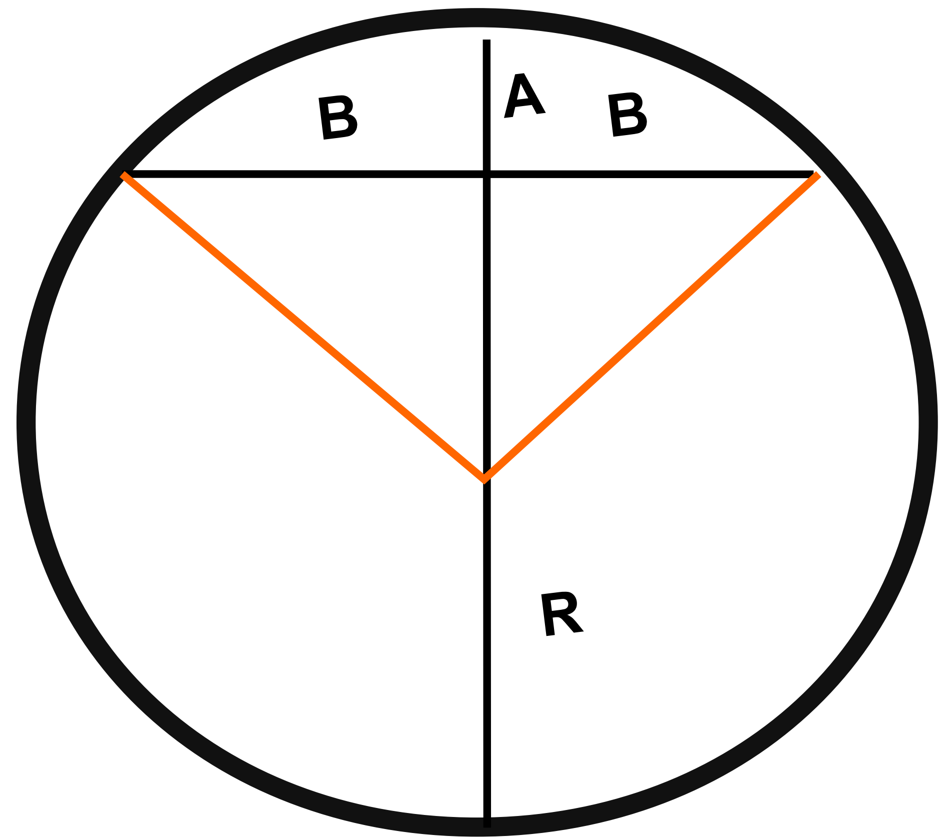




As c increases angle a approaches 90^0 and angle e approaches 0. Thus the tangent is perpendicular to the radius.

From the previous proof, we see that $c=(b-d)/2$





a=a opp. Intersecting angles
b=b, c=c measured by same arcs

C/C1 = B1/B Similar triangles
B*C= B1*C1

Example:

$B^2+(R-A)^2=R^2$

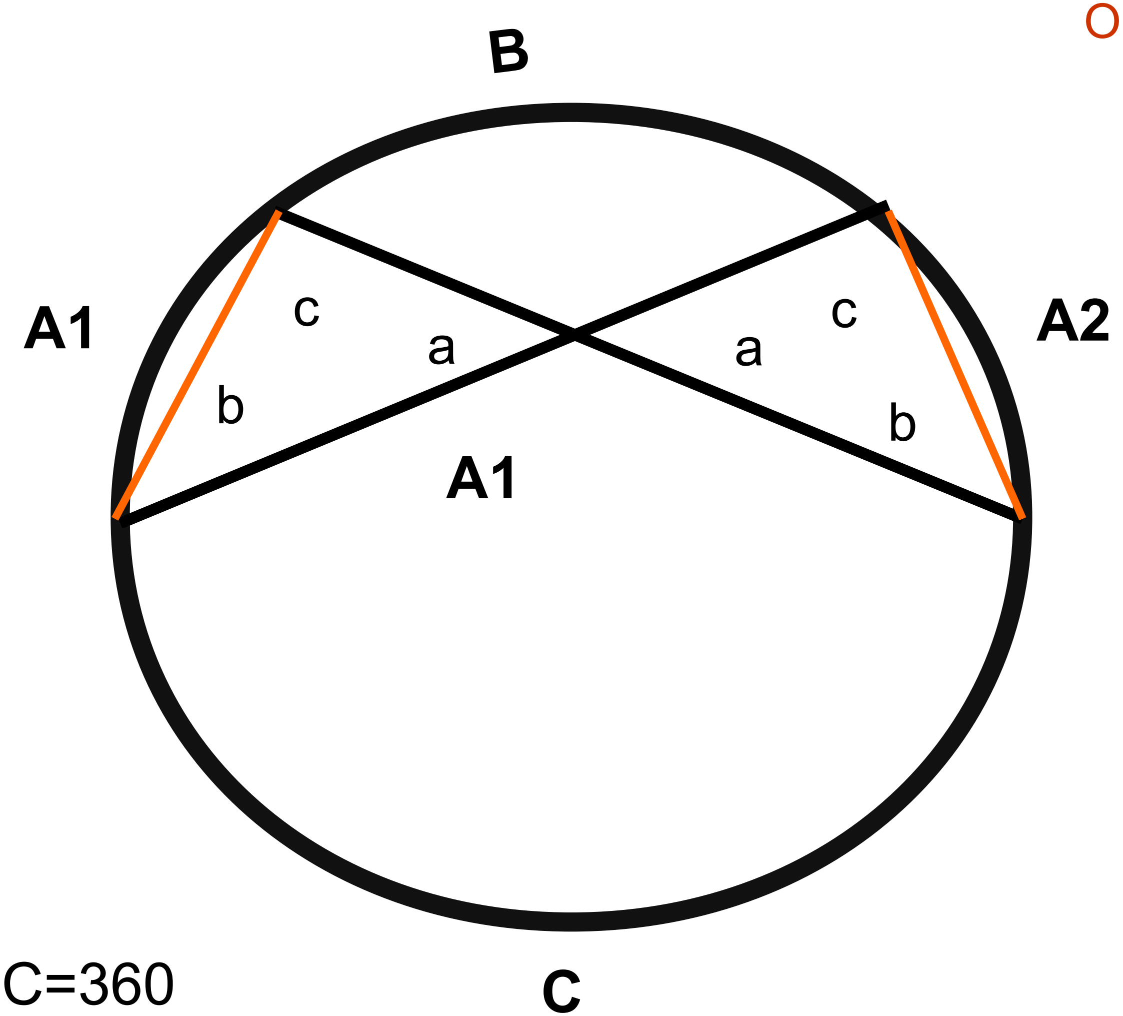
$B^2+R^2-2AR+A^2=R^2$

$A(A-2R)=-B^2$

$A(2R-A)=BXB=B^2$

Measure of angle inside a circle?

Optional lesson

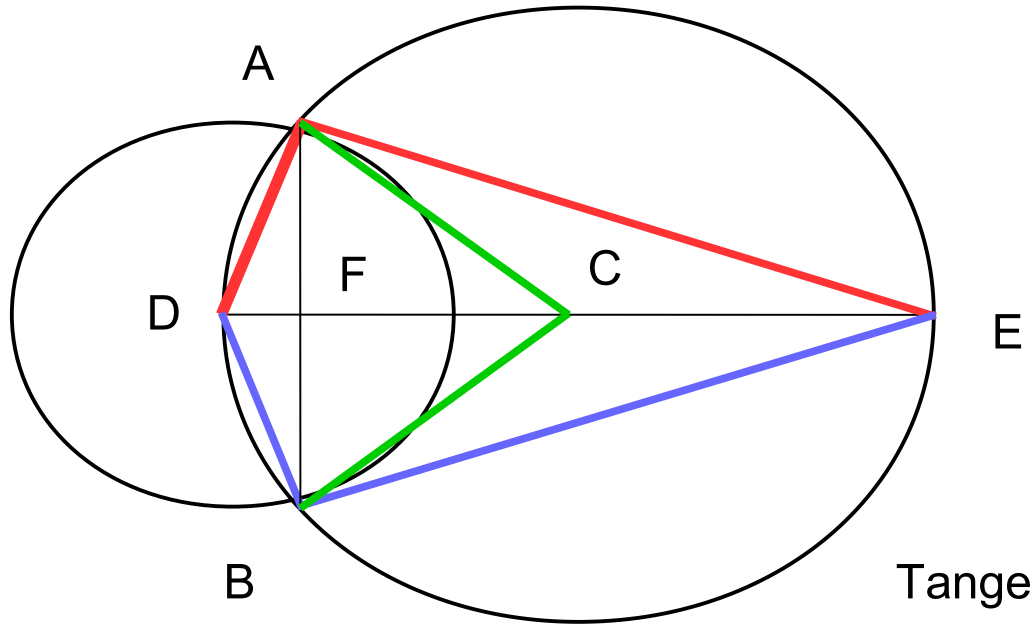


A1+A2+B+C=360
a+b+c=180
2a+2b+2c=2a+B+C=360
a=(A1+A2)/2

End of leson 5

Asking a question leads to discovery

Inscribed and Circumscribed Circles



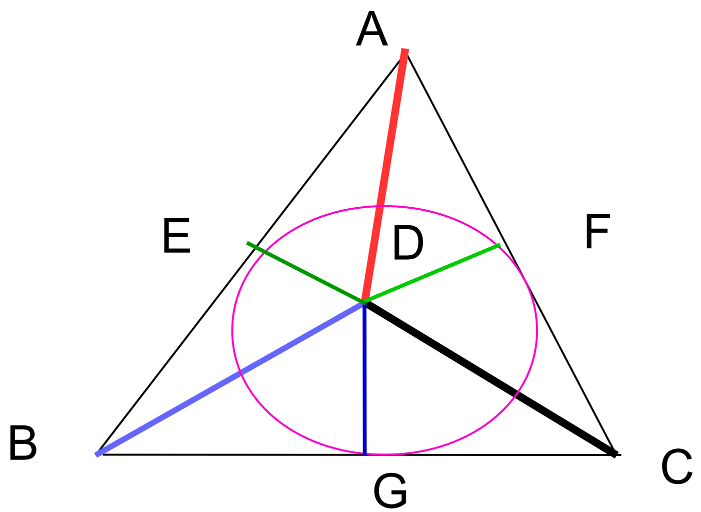
Tangent lines equal

Given: Circles centered at D and C
Prove: tangent AE=BE
DA⊥AE & DB⊥BE

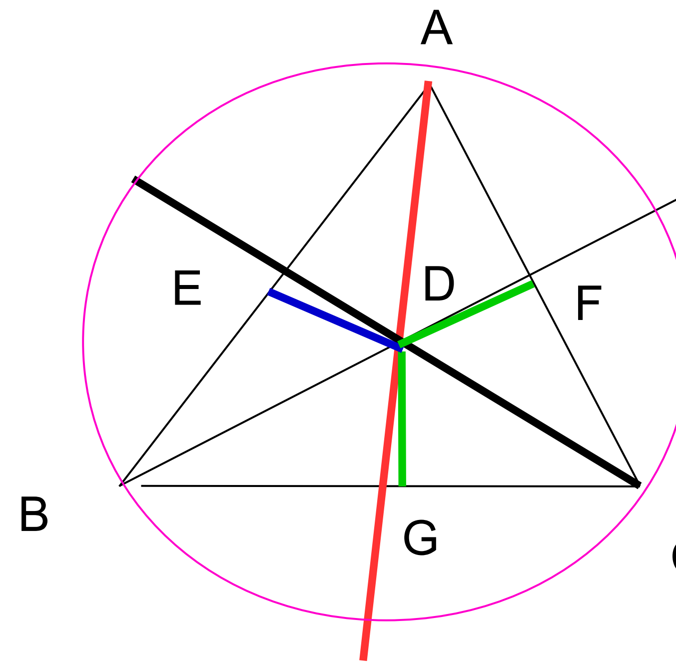
Proof:

Step	Reason
1. DA=DB	Radii
CA=CB	
2. DC=DC	Identity
3 ΔDAC≈ΔDBC	SSS
4.∠ ADE=∠BDE	Angles congr. Triangles
5. DE=DE	

Step	Reason
6. ΔDAE≈ΔDBE	SAS
7. AE=BE	Sides of congr. Triangles
8. DA⊥AE	Angle inscribed in semi-circle
DB⊥BE	



Bisect vertex angles (radii perpendic.)
Tangent lines equal



Bisect base (isosceles triangle)

Given: ΔABC, Angle Bisectors AD, AC
Perp. DE, DG, & DF
Prove: DE=DF=DG DB bisect ∠EBG

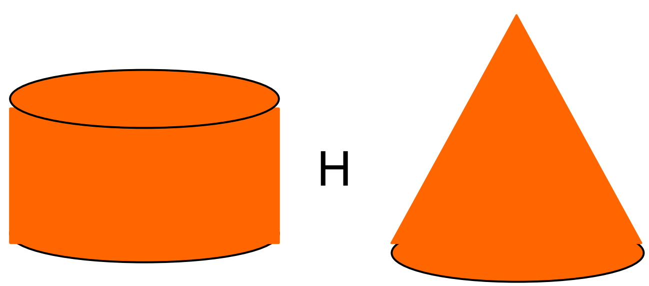
Proof; Step	Reason
1. ∠EAD=∠FAD	given
2. ∠ADE=∠AD	∠s of Δ=180^0
3 ΔEDA≈ΔFAD	ASA
4. DF=DE	sides of cong. Δs
5. DE=DF=DG	steps 1 to 4
6. BE=BG	tangent lines
7. Draw DB	constrruction
8. ΔDEB≈ΔDBG	SSS
9. BD bisects ∠EBG	congr. Δs

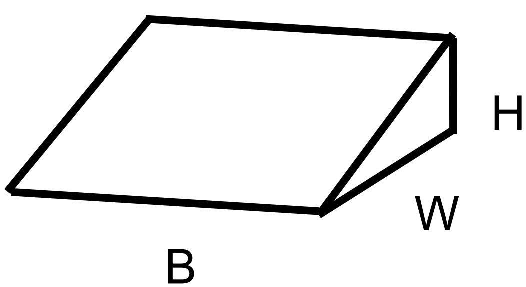
Given: ΔABC DG DF perp. Bisectors
Prove: AD=BD=CD DE perp. Bisect.

Proof: step	reason
1. AF=FC ∠AFD=∠CFD	given
2. DF=DF	identity
3. ΔFDA≈ΔFDC	SAS
4. AD=DC	sides of cong. Δs
5. AD=CD=BD	steps 1 to 4
6 Draw BE to midpoint	construction.
7. DE⊥AB	BDA isoceles Δ

Surface Area and volumes

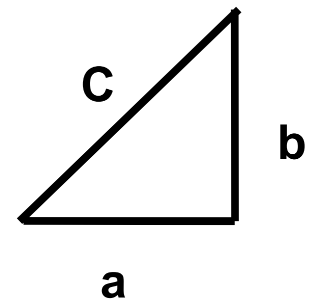
	Volume	Area
Rectangular Parallelpiped	BWH	$2(BW+BH+WH)$
Prism	$\frac{1}{2} BWH$	$WH+BH+HW+B(W^2+H^2)^{1/2}$
Cylinder	$\pi R^2 H$	$2\pi rH+2\pi r^2$
Pyramid (right angle)	$1/3$ Base H	$BW+B((W/2)^2+H^2)^{1/2}+W((B/2)^2+H^2)^{1/2}$
Cone (right angle)	$1/3 \quad \pi R^2 H$	$\pi r(r+(H^2+r^2)^{1/2})$
Sphere	$4/3 \quad \pi R^3$	$4\pi R^2$





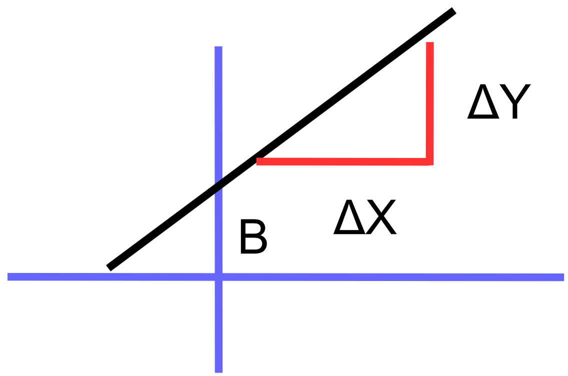
Trigonometry Slopes and intersections

Ratio of all sides of similar triangles are equal
Trigonometry is base on right triangles.



$\sin(\theta) = b/c$ $\cos(\theta) = a/c$ $\tan(\theta) = b/a$

$\csc(\theta) = c/b$ $\sec(\theta) = c/a$ $\text{cotan}(\theta) = a/b$

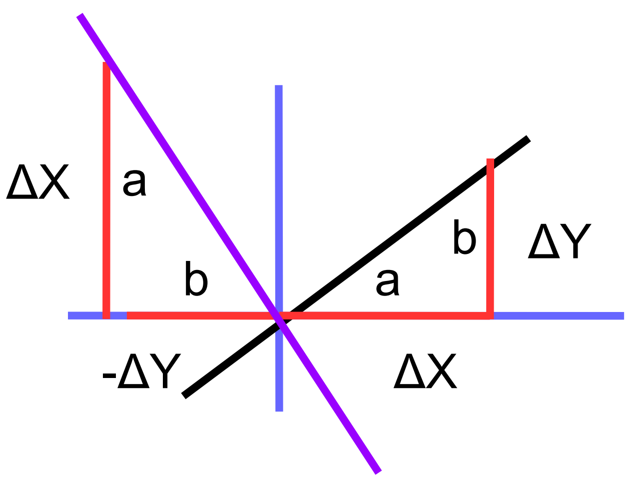


$m = \Delta Y / \Delta Y$ slope

$X=0,\ y=B$ intercept

$y = mX + B$

Slope of perpend.
Is negat. recipro.



$a+b=90^0$

$mp = \Delta X / -\Delta Y$
$= -1/m$

Intersecting lines

$Y1 = m1\ X + B1$

$Y2 = m2\ X + B2$

$y1 = y2$

$m1\ X + B1 = m2\ X + B2$

$X(m1-m2) = B2-B1$

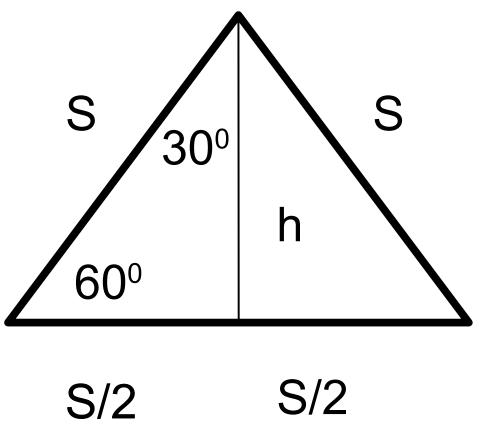
$x = (B2-B1)/(m1-m2)$

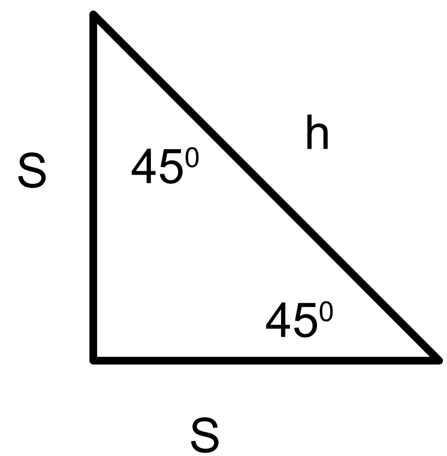
$m1=m2$ lines are parallel

Triangles and Angles

Triangles:
Equilateral, Isosceles, Scalene; Acute, right angle, obtuse

Angles:
Acute, right, obtuse, straight, reflex



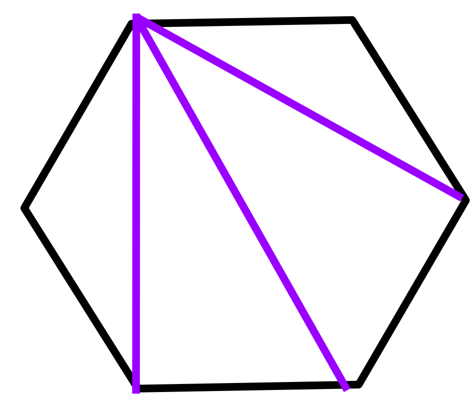


$h^2=s^2-(s/2)^2$
$H=3^{1/2}/2\ s$
$\sin(30^0)=s/2\ /s\ \ =1/2$
$\tan(30^0)=s/2\ /\ h\ =3^{1/2}/3$

$h^2=s^2+s^2$
$h=2^{1/2}\ s$
$\sin(45^0)=s/h=\ 2^{1/2}/2$
$\tan(45^0)=s/s=\ 1$

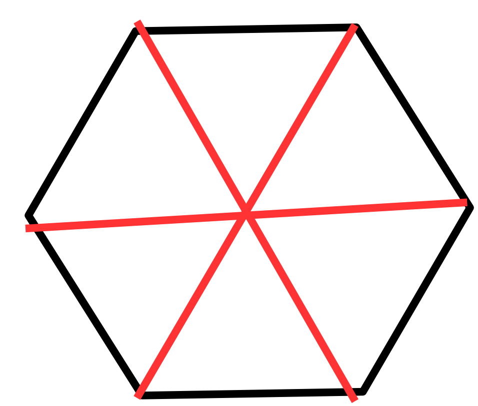
Sum of the angles of a polygon

First case;
No triangle = 2 less than the number of sides
The sum of the angles of a triangle=180^0



S=(n-2)x180

Second case:
The sum of the angles of about the center
= 360^0



S=180 n -360=(n-2)x180

The measure of an angle of a regular
Polygon is 180x(n-2)/n

Examples: triangle: 180x(3-2)/3=60
Square: 180(4-2)4=90

End of leson 6

Exam (goal=60 min.)

1.Define: st. line, parallel lines, similar triangles, and congruence
2. Prove that parallel lines never meet.

3. Create the family tree for quadrilaterals and prove that lines of a parallelgram are ||.
4. Find the general formula for the areas of geometric figures
.
5. Find the angle of a regular octagon
6. Name the different angles and triangles
7. Prove that the exterior angle of a triangle is the sum of the opposite interior angles.
8. Prove SAS

9. Name the different parts of a circle.
10. Illustrate how to determine the value of pi
11. Prove that an exterior angle of a circle is measured by half the measure of the difference of two intersecting arcs.
12. Draw a tangent to a circle from a point outside the circle, and prove that it makes a r right angle with the radius

13. Prove the pythagoren theorem two ways
14. Find area of an equilateral triangle of side of 3 inches.
15. Find $\sin(60^0)$ $\cos(60^0)$ $\tan(60^0)$

16. Prove slope of perpendicular to a line is the negative reciprocal of the slope of the line
17. Fine the point of intersection of lines: $3x+2y=12$ $4x - 3y=-1$
18. Find the slope of the line perpendicular to $4x-3y=-1$
19. Make a pyramid and cone from a sheet of paper.

Homework

Lesson 1

Bisect an angle

Copy an angle

Draw intersecting lines. Measure angles

From a point, draw a line perpendicular to a line

Draw a line parallel to a line, by copying an angle, Measure the angles

Lesson 2

Copy triangle using ASA, SAS, and SSS

Prove theorems for sum of angles of a triangle anf the bisector of an isosceles triangle

Measure angles from construction in these theorems

Draw a line twice the base of another line. Copy the angles of that triangle and measure the two new sides. (similar triangles)

Lesson 3

Prove why parallel lines never meet

Classify quadrilaterals

Construct a parallelogram, rectangle and dart

Lesson 4

Prove Pythagorean theorem two ways

Draw an octagon with sides of 1 in. Measure two diagonals calulate the ratio of the perimeter to eah of these diagonals (lesson for calculation pi)

Draw an inscribed angle and measure the central angle and inscribed angle

Draw an angle in a semi-circle. Measur that angle and several others drawn in that semi-circle.

Lesson 5
Prove the three circle theorems

Lesson 6
Make a cone and pyramid using a piece of paper
Draw a graph of a straight line using y-intercept and slope.
Draw a line perpendicular to that line and measur the slope of the perpendicular line.
Classify angles and triangles
Draw a 60-30 right triangle and fine the sin and tangent of 30^0 by measuring the sides and Calculating the ratios
Draw a regular pentagon and measure an interior angle. Calculate what that angle should Be from the formula.