Pascal Triangle

2	**1**
	3

1	**0**	**0**	**0**	**0**	**0**	
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1						

complete last 2 lines

$$(A+B)^0 = A^0B^0$$
$$(A+B)^1 = A^1B^0 + A^0B^1$$
$$(A+B)^2 = A^2B^0 + A^1B^1 + A^0B^2$$
$$(A+B)^3 = A^3B^0 + A^2B^1 + A^1B^2 + A^0B^3$$

$$(A+B)^0 = 1A^0B^0$$
$$(A+B)^1 = 1A^1B^0 + 1A^0B^1$$
$$(A+B)^2 = 1A^2B^0 + 2A^1B^1 + 1A^0B^2$$
$$(A+B)^3 = 1A^3B^0 + 3A^2B^1 + 3A^1B^2 + 1A^0B^3$$

Complete next 2 lines

$$(A+B)^4 =$$
$$(A+B)^5 =$$

Different Bases

$2A^2B^3 \times 3A^3BC^2 = 2 \times 3 \times A^2A^3B^3B^1C^2 = 6A^{2+3}B^{3+1}C^2 = 6A^5B^4C^2$

Distributive Property

$(2+3)x7 = 5x7=7+7+7+7+7=(7+7)+(7+7+7) = 2x7+3x7$

$(2+3)x(4+5) = 2x(4+5) + 3x(4+5)$

$= (2+3)x4 + (2+3)x5$

$2AB+3AB= 2(AB)+3(AB) = (2+3)(AB)= 5AB$

$$
\begin{array}{r}
A+B \\
\underline{\times A+B} \\
AA+AB \longrightarrow A^2+AB \\
BA+BB \longrightarrow \underline{\qquad AB+B^2} \\
(A+B)^2 = \qquad A^2+2AB+B^2
\end{array}
$$

$$
\begin{array}{r}
A^2+2AB+B^2 \\
\underline{\times \qquad A+B} \\
A^3+2A^2B+AB^2 \qquad\qquad \\
\underline{\qquad A^2B+2AB^2+B^3}
\end{array}
$$

$$(A+B)^3 = A^3+3A^2B+3AB^2+B^3$$

$$= 1A^3B^0+3A^2B^1+3A^1B^2+1A^0B^3$$

note $A^0=B^0=1$

Numerical example

$(A+B)^3=A^3+3A^2B+3AB^2+B^3$

$A=2 \quad B=3$

n	n^2	n^3
2	4	8
3	9	27
5	25	125

$(2+3)^3=2^3+3x2^23+3x2x3^2+3^3$

$125 = 8+3x4x3+3x2x9+27$

$= 8+36+54+27$

$= 44+81$

$= 125$

Factorials

$4! = 4x3!$
$3! = 3x2!$
$2! = 2x1!$
$1! = 1$
$1! = 1x0!$

$4! = 4x3! = 4x3x2! = 4x3x2x1! = 4x3x2x1 = 24$
$3! = 3x2! = 3x2x1! = 3x2x1 = 6$
$2! = 2x1! = 2x1 = 2$
$0! = 1!/1 = 1/1 = 1$

$$\frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5x4x3!}{2x3!} = 5x4/2 = 10$$

$3!/(3!0!) = 3!/(0!3!) = 1 \quad 3!/(2!1!) = 3!/(1!2!) = 3x2!/(2!1) = 3/1 = 3$

$1A^3B^0 + \quad 3A^2B^1 + \quad 3A^1B^2 + \quad 1A^0B^3 =$

$$\mathbf{(3!/(3!0!)A^3B^0 + (3!/(2!1!))A^2B^1 + (3!/(1!2!))A^1B^2 + (3!/(0!3!))A^0B^3}$$

Given:

$$e^x = x^0/0! + x^1/1! + x^2/2! + x^3/3! + x^4/4! + \ldots$$

x=	1	1/2	1/4	
0!	**1.0000000**	**1.00000 …**	**1.00000**	
1!	1.00000000 ...	.500000	.2500000	=1./4
2!	.50000000	.125000	.0312500	=.5/16
3!	.16666666	.020833	.0026041	=.1666.../64
4!	.04166666	.002604	.0001627	=.04166.../256
5!	.08333333	.000260	.0000081	=.0833.../1024
6!	.00138888	.000021	**1.28402**49	
7!	.00019841	.000001		
8!	.00002480	**1.64872**7		
9!	.00000275			
10!	.00000027			
	2.71828156			

$$e^x = x^0/0! + x^1/1! + x^2/2! + x^3/3! + x^4/4! + \ldots$$

$$e^y = \underline{y^0/0! + y^1/1! + y^2/2! + y^3/3! + y^4/4! + \ldots}$$

$$y^0x^0/0!0! + y^0x^1/0!1! + y^0x^2/0!2! + y^0x^3/0!3! + y^0x^4/0!4! + \ldots$$

$$y^1x^0/1!0! + y^1x^1/1!1! + y^1x^2/1!2! + y^1x^3/1!3! + y^1x^4/1!4! + \ldots$$

$$y^2x^0/2!0! + y^2x^1/2!1! + y^2x^2/2!2! + y^2x^3/2!3! + \ldots$$

$$+$$

$$(y^0x^0/0!0!) + (y^0x^1/0!1! + y^1x^0/1!0!) + (y^0x^2/0!2! + y^1x^1/1!1! + y^2x^0/2!0!) + \ldots$$

$$= (x+y)^0/0! \quad + (x+y)^1/1! \quad + (x+y)^2/2! \quad + \quad \ldots$$

$$= e^{x+y}$$

note: $(x+y)^2/2! = (2!y^0x^2/0!2! + (2!y^1x^1/1!1! + 2!y^2x^0/2!0!)/2!$

$$= (y^0x^2/0!2! + y^1x^1/1!1! + y^2x^0/2!0!)$$

ReCap

$$e^x \times e^y == e^{x+y}$$

$$e^x = x^0/0! + x^1/1! + x^2/2! + x^3/3! + x^4/4! + \ldots$$

$$e^y = \underline{y^0/0! + y^1/1! + y^2/2! + y^3/3! + y^4/4! + \ldots}$$

$$(y^0x^0/0!0!) + (y^0x^1/0!1! + y^1x^0/1!0!) + (y^0x^2/0!2! + y^1x^1/1!1! + y^2x^0/2!0!) + \ldots$$

$$= e^{x+y}$$

$$(x+y)^1/1! = (y^0x^1/0!1! + y^1x^0/1!0!)$$

$$(x+y)^2/2! = (y^0x^2/0!2! + y^1x^1/1!1! + y^2x^0/2!0!)$$

$$(x+y)^3/3! = (y^0x^3/0!3! + y^1x^2/1!2! + y^2x^1/2!1! + y^3x^0/3!0!)$$

Negative Exponents

$(-1)^{odd}=-1 \quad (-1)^{even}=1$

$e^{-x} = x^0/0! - x^1/1! + x^2/2! - x^3/3! + x^4/4! - \ldots$

$e^{-1/4} = 1 - .25 + .031250 - .002604 + .000162 = .77880\ldots$

$1/e^{1/4} = 0.77880$

Logarithms

$A^{\log A\,(B)} = B$ definition

$2 \times 2 \times 2 = 2^3 = 8$

$2^{\log 2(8)} = 8 = 2^3$

$\log_2(8) = 3$

$\log_2(4x8) = \log_2(4) + \log_2(8)$

$5 = 2 + 3$

$\log_2(4^3) = 3x\log_2(4)$

$6 = 3 \times 2$

$\log_2(1) = 0 \quad 2^0 = 1$

$\log_2(2) = 1 \quad 2^1 = 2$

$\log_2(4) = 2 \quad 2^2 = 4$

$\log_2(8) = 3 \quad 2^3 = 8$

$\log_2(16) = 4 \quad 2^4 = 16$

$\log_2(32) = 5 \quad 2^5 = 32$

$\log_2(64) = 6 \quad 2^6 = 64$

$A^{\log A\ (B)} = B$ definition

$A^{\log A(BxC)} = BxC = A^{\log A(B)} \times A^{\log A(C)} = A^{\log A\ (B) + \log A\ (C)}$

$\log_A(BxC) = \log_A (B) + \log_A (C)$

$A^{\log A(B\text{^}C)} = B\text{^}C = (A^{\log A\ (B)})^C = A^{C \log A\ (B)}$

$\log_A(B^C) = C \log_A(B)$

$A^{\log A (B)} = B$ $B^{\log B (B)} = B = B^1$ $\log_B(B)=1$

$\log_B(A^{\log A (B)})=\log_B(B)$

$\log_A(B) \log_B(A)= 1$

$A^x= B^C$

$\log_A(A^x)= \log_A(B^C)$

$x \log_A(A)= C \log_A(B)$

$x= C \log_A(B)$

Finding Logarithm of a Number

$$10^{.301}=x \qquad 10^{y}=2$$

$$10^{.301}=10^{.25+.03125+015625+\ldots+0}=10^{.25}x10^{.03125}x10^{.015625}x\ldots x10^{0}$$

n	10^n	.301	1	2	0
2	100	-.25	x1.778279	/1.778279	+. 25
1	10	.051	1.778279	1.124682	.25
.5	3.162277	-.03125	x1.074607	/1.074607	+.03125
.25	1.778279	.01975	1.910951	1.046599	.28125
.125	1.333521	-.015625	x1.036632	/1.036632	+.015625
.0625	1.154781	.004125	1.980953	1.009615	.296875
.03125	1.074607	-.003906	x1.009035	/1.009035	+.003906
.015625	1.036632	.000219	1.998850	1.000574	.300781
.0078125	1.018151	-.000122	x1.000281	/1.000281	+.000122
.00390625	1.009035	.000097	1.999411	1.000293	.300903
.001953125	1.004507				
.0009765625	1.002251		x=1.999411	$y=\log_{10}(2)=.301$	
.00048828125	1.001124				
.000244140625	1.000562				
.0001220703125	1.000281				

```python
def sqrt(xi):
    x=3
    i=0
    while i<6:
        #print(x,i)
        x=.5*(x+xi/x)
        i=i+1
    return x

def logb(b,xi):
    x=xi
    pb=b
    i=0.
    while b<x:
        i=i+1.
        x=x/b
    k=0
    p2=1.
    ans=0.
    while k<54:
        if pb<x:
            ans=ans+p2
            x=x/pb
        p2=p2/2.
        pb=sqrt(pb)
        k=k+1
    return ans+i
```

```python
def main():
    i=1.
    print  "n    log10(n)"
    while i<11:
        a=logb(10,i)
        print i,a
        i=i+1
    return
main()
```

```
n     log10(n)
1.0    0.0
2.0    0.301029995664
3.0    0.47712125472
4.0    0.602059991328
5.0    0.698970004336
6.0    0.778151250384
7.0    0.845098040014
8.0    0.903089986992
9.0    0.954242509439
10.   1.0
```

Generating Logarithm Tables

$\log(A)=\log_{10}(A)$

$\log(2)=.301$ $\log(3)=.477$ $\log(5)=\log(10/2)=\log(10) - \log(2)=1-.301=.699$

$\log(48)=\log(2^4x3)=4 \log(2)+\log(3)=4x.301+.477=1.204+.477=1.681$

$\log(50)=\log(10x5)=\log(10) + \log(5)=1.699$

$\log(49)\approx(\log(48)+\log(50))/2=(1.681+1.699)/2=3.380/2=1.690$

$\log(49)=\log(7^2)=2 \log(7)\approx1.690$ $\log(7)\approx1.690/2=.845$

$\log(1)=0$

$\log(2)=.301$

$\log(3)=.477$

$\log(4)=\log(2^2)=2 \log(2)=2x.301=.602$ $\log(A^B)=B \log(A)$

$\text{Log}(5)=.699$

$\log(6)=\log(2x3)=\log(2)+\log(3)=.301+.477=.778$ $\log(AxB)=\log(A)+\log(B)$

$\log(7)=.845$

$\log(8)=\log(2^3)=3 \log(2)= 3 x..301=.903$ $\log(A^B)=B \log(A)$

$\log(9)=\log(3^2)= 2 \log(3)= 2 x .477=.954$ $\log(A^B)=B \log(A)$

$\log(10)=1.$

$\log(7)\approx(\log(8)+\log(6))/2=(.903+.778)/2=1.681/2=.841$ not very close

Multiple Multiplications

$$(1.01)^{100} = 1.01^{64+32+4} = 1.01^{64} \times 1.01^{32} \times 1.01^{4} \quad A = 1.01$$

100
-64
36
-32
4
-4
0

A^{2}
1.01
x1.01
101
101
1.0201

A^{4}
1.0201
x1.0201
10201
20402
10201
1.04060401

A^{8}
1.0406
x1.0406
62436
41624
10406
1.08284836

A^{16}
1.0828
x1.0828
8662
21656
86624
10828
1.17245584

A^{32}
1.1725
x1.1725
58625
23450
82075
11725
11725
1.37475625

A^{64}
1.3748
x1.3748
109984
54992
96237
41244
13748
1. 89007504

$A^{64} \times A^{32}$
1.3748
x1.8901
13748
123732
109984
13748
2.59850948

e=2.718281828...

$A^{96} \times A^{4}$
2.5985
x1.0406
155910
103940
25985
2.70399910 actual=2.70481...

Generating Exponential Series e^x

$(A+B)^3=3!/(3!0!)A^3B^0 + 3!/(2!1!)A^2B^1 + 3!/(1!2!)A^1B^2 + 3!/(0!3!)A^0B^3$

$$(A+B)^3=\sum_{m=0}^{3} 3!/((3-m)!m!)\ A^{3-m}B^m$$

m	$3!/((3-m)!m!)\ A^{3-m}B^m$
0	$3!/((3-0)!0!)\ A^{3-0}B^0 = 3!/(3!0!)A^3B^0$
1	$3!/((3-1)!1!)\ A^{3-1}B^1 = 3!/(2!1!)A^2B^1$
2	$3!/((3-2)!2!)\ A^{3-2}B^2 = 3!/(1!2!)A^1B^2$
3	$3!/((3-3)!3!)\ A^{3-3}B^3 = 3!/(0!3!)A^0B^3$

$$(A+B)^n=\sum_{m=0}^{n} n!/((n-m)!m!)\ A^{n-m}B^m$$

m	$n!/((n-m)!m!)$	n=3
0	$n!/((n-0)!0!)= n!/(n!0!) = 1/0! =1$	1
1	$n!/((n-1)!1!)= n\ (n-1)!/((n-1)!1!)= n/1!$	3/!=3
2	$n!/((n-2)!2!)= n\ (n-1)(n-2)!/((n-2)!2!= n(n-1)/2!$	3x2x1/2!=3
3	$n!/((n-3)!3!)= n\ (n-1)(n-2)(n-3)!/((n-3)!/3!)= n(n-1)(n-2)/3!$	3x2x1/6=1
3	$n!/((n-4)!4!)= n(n-1)(n-2)(n-3)/4!$	3x2x1x0/24=0

$$(A+B)^n=\sum_{m=0}^{\infty} n!/((n-m)!m!)\ A^{n-m}B^m$$

$$A=1 \quad (A+B)^n=\sum_{m=0}^{\infty} n!/((n-m)!m!)\ A^{n-m}B^m=\sum_{m=0}^{\infty} n!/((n-m)!m!)\ B^m=(1+B)^n$$

$x=nB$ therefore, $n=x/B$

$$\begin{aligned}(1+B)^{x/B} &= B^0/0! + nB^1/1! + n(n-1)B^2/2! + n(n-1)(n-2)B^3/3! + \ldots\\ &= B^0/0! + nB/1! + n(n-1)BB/2! + n(n-1)(n-2)BBB/3! + \ldots\\ &= B^0/0! + nB/1! + BnB(n-1)/2! + BnB(n-1)B(n-2)/3! + \ldots\\ &=1/0! + nB/1! + nB(nB-B)/2! + nB(nB-B)(nB-2B)/3! + \ldots\end{aligned}$$

$$((\mathbf{1+B})^{\mathbf{1/B}})^x = 1/0! + x/1! + x(x-B)/2! +x(x-B)(x-2B)/3!$$

$\lim B\rightarrow 0$

$$\mathbf{e^x} = 1/0!+ x/1! + x^2/2! + x^3/3! +$$

$$= x^0/0!+ x^1/1! + x^2/2! + x^3/3! +$$

With the binomial expansion

$(1.01)^{100} = (1 + .01)^{100}$
$= 1+100x.01/1!+100x99x.01^2/2!+100x99x98x.01^3/3!$
$= +100+99x98x97x.01^4/4!+100x99x98x97x96x.01^5/5+...$
$=2.7034$

Note: $100x99x98x.01^3/3!=(100x99x.01^2/2!)x98x.01/3$