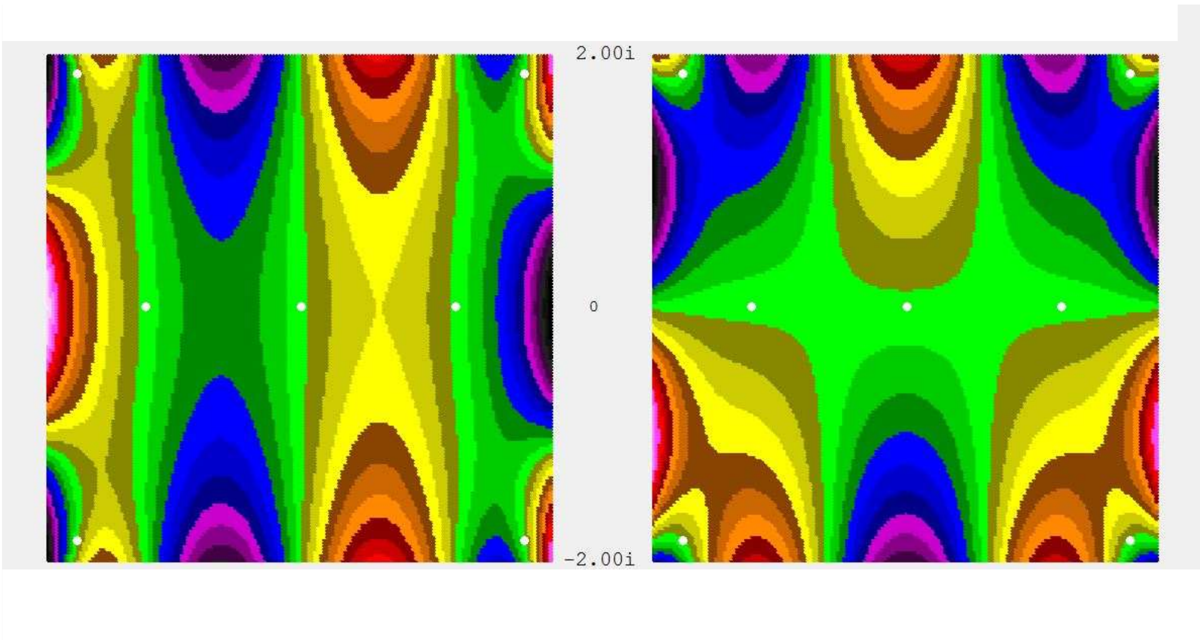
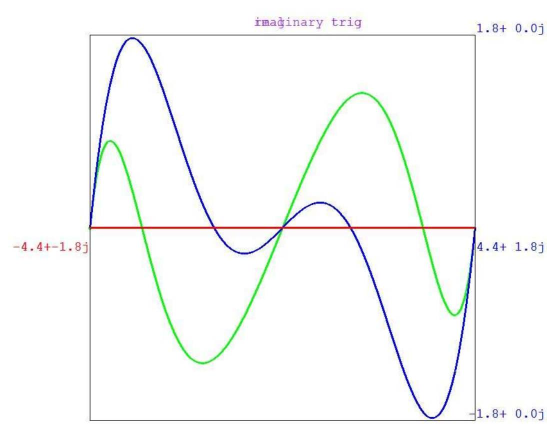
Mathematics Reinterpreted





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Abstract

Is math is founded upon interpretations that are nor rigorously challenged? There are many misconception and anomalies that can be explained by more rigorous interpretations. Zero being considered nothing, negative numbers not be counting numbers, addition and multiplication being considered commutative, a line not having sides, an area not being negative are just a few examples of interpretations that should be reconsidered. However, thinking of a number as a vector, one can invalidate all of these axioms. We will show the thought processes in challenging these interpretations and the removal of some anomalies resulting with existing interpretations.

Motivation

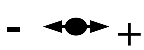
Among some of the early motivations for questioning the foundations of mathematics was trying to understand how factoring of the infinite series for trigonometric functions could be used to determine the value of pi. We will discuss this problem later in more detail. We can organize mathematics to make it easier to understand and remember:

	addition	multiplication	exponentiation
Inverses	subtraction	division	logarithms
geometry	translation	scaling	rotation
numbers	negative nos.	fractions	irrational nos.
spec nos.	0	1	√-1
standard	point	length	angle
vector	position	magnitude	direction

The strategy is to discuss a different way to talk about axioms to be challenge and then to show the inconsistencies in those axioms.

Zero

Zero is the start of the positive (+0) and negative numbers (-0), has no magnitude, and is represented by a dot with vectors going in the opposite directions. This interpretation implies that –0 and +0 occupy the same position since zero has no magnitude. This becomes the definition of a 'point' in geometry.



We use the function y=1/x when x=0 as illustration of this interpretation since at –0, we have -∞ and at +0, we have +∞. In computer printouts, a number very close to zero is written as –0 on the left side and +0 on the right side of zero.

According to a Google search, the statement that "0 is nothing" is a philosophical interpretation, while the existence and properties of zero are established within mathematics through a system of axioms. Zero is a core concept that is fundamental to modern mathematics. Yes, zero often represents "nothing" as the absence of quantity (like zero apples), but it's also a vital number in math, acting as a placeholder (in 205), an additive identity (x+0=x).

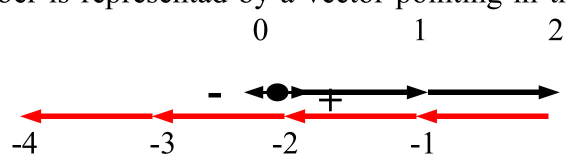
Every number as one is in 215 is a placeholder. Zero makes numbers bigger when placed before the decimal point and smaller one place after the decimal point as in this special example 2.5, 21.5 and 2.15. In the multiplicative identity x*1=x, 1 is not nothing because it does not change the value.

A favorite application is the use of zero on a thermometer. On the Farenheit and Celcius scales, the zero does not indicate lack of heat; only on the Kelvin scale does it do so.

All this false interpretation of 0 as nothing is distracting from the real value of zero in making arithmetic operations easy to perform and getting correct answers more reliably.

Negative numbers

A negative number is representad by a vector pointing in the opposite direction of a positive number.



We can see that 2+ -4 = -2 and that 2+ -2=0. From this we can derive the definition of a negative number and th rules of operation: The definition is as follows: When a negative number is added to its positive inverse, we get the identity element for addition. A negative sign is an indication of direction. Thus a negative sign changes the direction of a positive number to a negative number and a negative number to a positive number:

This concept is a interpretation so later we will produce examples with multiplication to illustrate that it is consistent with our current knowledge of mathematics. Let us look some proofs:

-2+2=0	definition	
2= 0 – -2	subtraction	confirms interpretation of - sign
-2= 0 –2	subtract	explains notation of a negative number

We can see that 0+0=0 which implies that 0=0-0
We note from above that -2=0-2
Replacing 2 with zero we have: -0=0-0
We can see how we are lead to believe that 0 and –0 occupy the same place on the number line.
We have started with negative numbers to illustrate properties of zero.

Let us start with 0. An integer is created by adding 1 to the previous number. Lets look at our number line:

B A 0 1 2 3

	B+1=A	create A
	B+1+1=A+1	add one
* A+1=0	B+2 =0	substitute 0 for A+1
A =0-1	B =0-2	subtract
A = -1	B =-2	notation
-1 +1=0	-2+2=0	substituting for A and B

These exercises give us more confidence in our new interpretation of negative numbers. We also see that it makes it easier to do computations.

Counting Numbers

We are going to see how this new way of interpretation, allows to make math easier to learn and apply our knowledge. Traditionally, counting numbers are only positive integers greater than zero. Yet, zero and negative number can be intepreted to be counting numbers. Since multiplication is a counting process, let us look at the impact of negative numbers on multiplication. The expression 3x2=0+2+2+2, and 3x-2=0-2+-2+-2=0-(2+2+2)=0-6=-6 When we add repeated positive numbers, we add then a place a plus sign in front of the result. When we add repeated negative numbers, we add them as positive numbers and place a minus sign in front of the results.

When we use a negative number as a counter, we add the numbers it the traditional way, but we place a minus sign in front of the result. A negative result becomes positive and a positive result becomes negative: -3 x –2 =0-(-2+-2+-2)=0 - -(2+2+2)=0+(2+2+2)=6
-3x2==0- +(2+2+2)=0-(2+2+2)=0-6=-6
This makes sense with the vector interpretation of numbers, but is difficult to understand from our traditional view of negative numbers. Note that in one step, you could prove that the product of negative numbers is a positive number.

Multiplication

The major impact of the reinterpretation of zero and negative numbers is upon multiplication. The convention definition of multiplication is 3x2=2+2+2. We are going to define it as: 3x2=0+2+2+2. This is done by pattern recognition:

3x2=2+2+2	3x2=0+2+2+2
2x2=2+2	2x2=0+2+2
1x2=2	1x2=0+2
0x2=?	0x2=2

In the first example we keep dropping a +2 but in the last step there is no +2 to drop.
We know that 2x0=0+0=0 or 2x0=0+0+0=0. Thus we have shown that multiplication is commutative with respect to zero without applying the commutative property.

Let us look at negative numbers 3x-2=0+-2+-2+-2=0-(2+2+2)= -6 and
-2x3=0+ -(3+3)=0-6= -6

We just used our rules for multiplying by negative numbers. Thus again, we do not have to use the commutative rule to show that the multiplication with negative numbers is commutative.

However, we still note that multiplication appears to be commutative. Now let us look at the distributive property.

5x1=1+1+1+1+1=5
5x3=5x(1+1+1)=5x1+5x1+5x1=5+5+5=3x5

It is the distributive property that makes multiplication appear to be commutative. Let us look at the following example:

(-4)^1=(-4)^2x1/2=((-4)^2)^1/2=16^1/2=4
(-4)^1=(-4)^1/2x2=((-4)^1/2)^2=(2i)^2=-4

Thus multiplication is not commutative for fractions. Let us look at matrix multiplication:

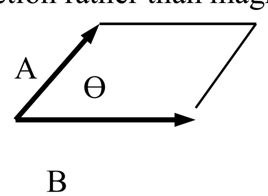
(1 2) x (5 6) = (19 22) (5 6) x (1 2) = (23 34)
(3 4) (7 8) = (43 50) (7 8) (3 4) = (31 46)

It is obvious that multiplication is not commutative for matrices. To resolve this anomaly, we make the following definitions. Multiplication obeys the distributive property. Multiplication is not commutative.

Thus, this subtle change in the definition of zero and negative numbers removes many of the anomalies created by the former interpretations.

Numbers are vectors

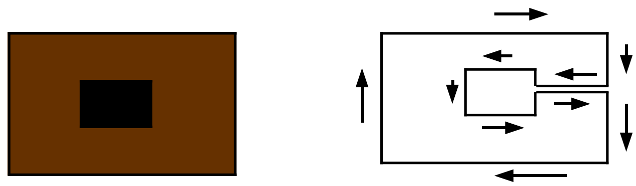
As we look at more of the anomalies in the current mathematical logic, we begin to realize, that if we think of numbers as vectors, this anomalies disappear as they did with the concept of the minus sign as an indicator of direction rather than magnitude.



The product of vectors A and B is $AB\sin(\Theta)$ which is the magnitude or area of the parallelogram. If A=B and $\Theta=90^0$ we have the area of a unit square. Thus traditional multiplication assumes orthognality of the width and length.

According to tradition, the area cannot be negative and a line does not have sides. With a number as a vector the line has a right side (left) and a left side.When we calulate the distance from the line to a point, the left side gives us a positive answer an the right side a negative answer.This leads the perimeter to hace a direction of clockwise or counterclockwise.

The following diagram illustrates these points:



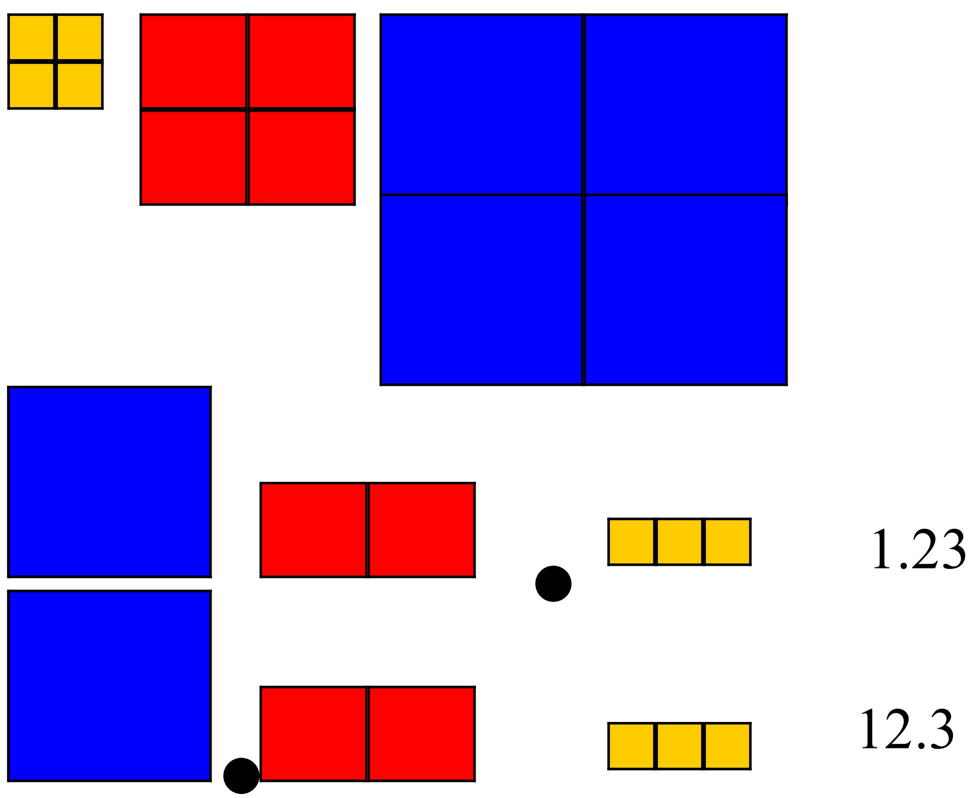
For the outer rectangle, we go clockwise and the inner rectangle we go counter clockwise. The inside of the outer rectangle is the right side of the lines. Thus the clock direction can be used to determine if an area is positive or negative. You should begin to notice these consistencies with the vector form of a number.

Fractions

Fractions are defined similary to the way negative numbers are defined. When a fraction is multiplied by its multiplicative inverse, the result is the identity element for multiplication:

$$2x1/2=1/2 + 1/2=1$$

Where as addition is a translation, multiplication is a magnifiation. Mathematics does not define a basic size, but fractions identifies the problem. Let us look at fractions, in base 4.



If the blue square is one unit the red square is 1/4 of it, and the yellow square is 1/16 of the blue square. If the red square is one unit, the blue square is 4 units and the yellow square is 1/4 of the red square.

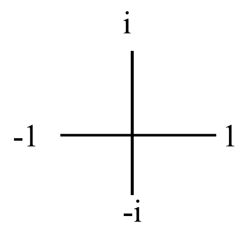
Thus $1.23= 1+2/4+3/16 = 1+11/16=27/16$
And $12.3= 1*4+ 2x1+3/4=6+3/4=27/4$

Since this was base four, we had to multiply by 4 to move the decimal point one position to the right. You can see why one has difficulty understanding fractions; partly due to the fact that the definition of length is arbitrary.

Complex Numbers

Once we define a number as a vector, understanding complex numbers becomes much easier to do. It is through exponention that we discover complex numbers: $4^{1/2}=(2x2)^{1/2}=2$
$4^{1/2}=(-2x-2)^{1/2}=-2$
$(-4)^{1/2}==2\sqrt{-i}$

The we discover : $i^2=i*i=-1$
$i^3=i*i*i=(i*i)*i=-i$
$i^4=(i*i)*(i*i)=-1x-1=1$



We can see from the graph and the algebra, that every time we multiply by 'i', we rotate 90^0 (orthogonality). We also discover that $e^{i\Theta}=\cos(\Theta)+i*\sin(\Theta)$. Thus complex numbers introduce both rotation and dimension.

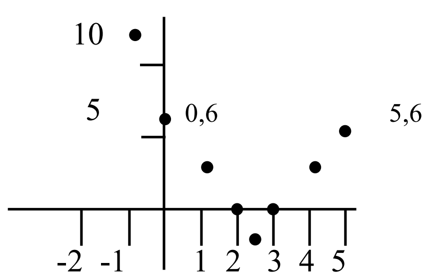
Roots of a polynomial.

The problem starts with what we mean by roots of a polynomial.

$Y=(x-2)*(x-3)=x^2-5x+6$. The roots of that polynomial are 2 and 3 because it is at these points that the curve crosses the x=0 axis. Let us build a table and plot a graph.

$y= x^2-5x+6$

x	y
-1	12
0	6
1	2
2	0
.5	.25
3	0
4	2
5	6



Let us now consider another function.

y=(x-(1+i))*(x-(1-i))=x^2-2x+2

y = x^2-2x+2

x	y
-2	10
-1	5
0	2
1	1
2	2

→

x	-2	-1	0	1	2
2i	6-12i	1+8i	-2-4i	-3+0i	-2+4i
1i	9-6i	4-4i	1-2i	**0+0i**	1+2i
0i	10+0i	5+0i	2+0i	1+0i	2+0i
-1i	9+6i	4+4i	1+2i	**0+0i**	1-2i
-2i	6+12i	1-8i	-2+4i	-3+0i	-2-4i

First we look at substituting real values in the equation. This gives us real values for “y” which match the middle blue row in the second table. The entries in the second table are result of putting a complex value for x. The top row represents the real component and the vertical row represents the imaginary component of y . Thus, f(2+2i)=-2+4i. We also see that there are only two places where both the real part and the imaginry parts are both 0. This is the plain where y=0 intersects the curves.

x	y: -2	-1	0	1	2
2i	6	1	-2	-3	-2
1i	9	4	1	**0**	1
0i	10	5	2	1	2
-1i	9	4	1	**0**	1
-2i	6	1	-2	-3	-2

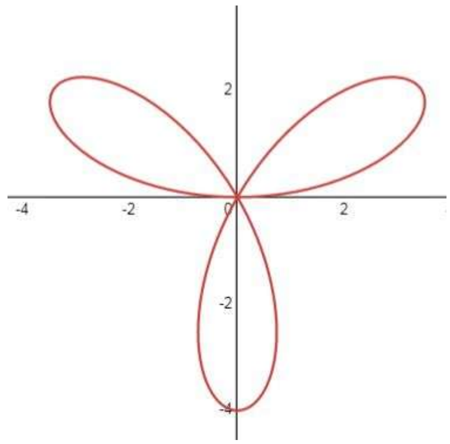
yi: -2	-1	0	1	2
-12i	8i	-4i	0i	4i
-6i	-4i	-2i	0i	2i
0i	0i	0i	0i	0i
6i	4i	2i	0i	-2i
12i	-8i	4i	0i	-4i

In these two tables we can look at the vertical rows (real plains) and the horizontal rows (imaginary plane) to see a parabolic cross section. This is easier to see if we do this in conjunction with contour plots.

This allows us to interpret the complex number as a vector. We are familiar with the three spatial dimensions, We now see six. I have not yet figured out what this does to the dimension of time. Perhaps, there is a way to go faster than the speed of light. On the series ‘Star Trek’, they do reference warp speed (faster than the speed of light). Anti matter was theorized by Paul Dirac through the use of ‘imaginary’ numbers.

Calculus

Besides introducing rotation and dimension, exponentiation introduces irrational numbers — numbers that can not be represented by fractions. This also leads us to the concept of continuity. Look at the curve for 4sin(3Ø):



A curve is initially plotted with a few points to give us a feel for the shape of the curve. As we add more and more points the curve takes on a more solid look. We then see crossings at the origin. We can see three arcs intersecting at the origin. To distinguish these arcs we use calculus. The slope of the arc going into the intersection should match that of the arc coming out of the intersection. If they do not match, we consider the curve discontinuous at that point. This simple interpretation brings in a new area or concept of math. We also see how the topics of angles and continuity begin to be ordered.

Measurement and visualization

What is length, area, or angle. They are represented by standards. The length is some arbitrary measurement upon which people agree: 1 inch, 1 yard, or 1 meter. For area, it is a length and angle that upon which people agree. For angles, we chose there to be 360^0 in a circle possibly based on a year being approximately 360 days. Here is an answer from Google: Mathematics itslf is self-sufficient without units. The "objects" of mathematics are complete abstractions that are rigorously regulated by sets of agreed upon axioms from which theorems can be proven, As far as mathematics goes, strictly speaking, that's all. The AI support in google took a while to answer this question.

Summary

By reinterpreting the concepts of zero and negative numbers, we were able to explain the anomalies of traditional math. The key was to recognizing that numbers are best represented by vectors rather than scalars. While we could completely replace that which was taught, it is best to leave some of those lessons because we show how to challenge a new concept giving us greater confidence in our ability.

The understanding of mathematics allows to solve problems. For example, it has been stipilated that a line has no sides. Several decades back I received a patent base upon a line having sides. The patent showed how one could thicken lines to make a more attractive street map.
In physics, we have determine absolute zero for measuring heat content is -459 degrees Fahrenheit. Zero degrees centigrade and zero degrees Fahrenheit are different markers on a heat measuring device (thermometer). Formulas in physics are based upon absolute zero. Negative tem-

peratures have no meaning in the measurement of temperature.

Even something as simple as number floors is suspect. In Europe the floor marked first is our second floor. Based upon the interpretaion in this paper the Europeans are correct.

Even within math itself. Addition has to be define as being commutative for finite sums, not for infinite sums.

Appendix

Let us look at how a problem evolves. We start off checking the factorization of the infinite series for the sine to evaluate the value of pi. First we look at the equation:

y=-x^7/7!+x^5/5!-x^3/3!+x\1!

This has seven roots; two complex pair that are negative and positive. There are three real roots; none of them multiples of pi.

(-4.43400+1.84375j), (-4.43400-1.84375j)
-3.0786423044815145, 0.0, 3.078642304481513,
(4.43400+1.84375j), (4.43400-1.843752j)

We can see five of the seven crossings. We had expected to see something of this nature: x(1-(x/π)^2)(1-(x/2π)^2)((1-(x/3π)^2) which did not match:

-x^/7!+x^5/5!-x^3/3!+x^1/1!

We then tried some more expansions of the sine series:

5 roots (1 real)
roots: -3.236854+0.690815j, -3.236854-0.690815j,
0,
3.236854+0.690815j, 3.236854-0.690815j

7 roots (3 real)
roots: -4.434005+1.843752j, -4.434005-1.843752j,
-3.078642, 0, 3.078642,
4.434005+1.843752j, 4.434005-1.843752j

9 roots (5 real)
roots: -5.351342+3.148084j, -5.351342-3.148084j,
-4.963153, -3.148690, 0, 3.148690, 4.963153,
5.351342+3.148084j, 5.351342-3.148084j

11 roots (3 real)
roots: -6.193643-4.521216j, -6.193643+4.521216j, -5.759862+1.014514j, -5.759862-1.014514j
, -3.141148, 0, 3.141148,
5.759862+1.014514j, 5.759862-1.014514j, 6.193643+4.521216j, 6.193643-4.521216j

13 roots (5 real)
roots: -6.970576+5.953662j, -6.970576-5.953662j, -6.784141-1.993144j, -6.784141+1.993144j,
-5.978351, -3.141614, 0, 3.141614, 5.978351,
6.784141+1.993144j, 6.784141-1.993144j, 6.970576-5.953662j, 6.970576+5.953662j

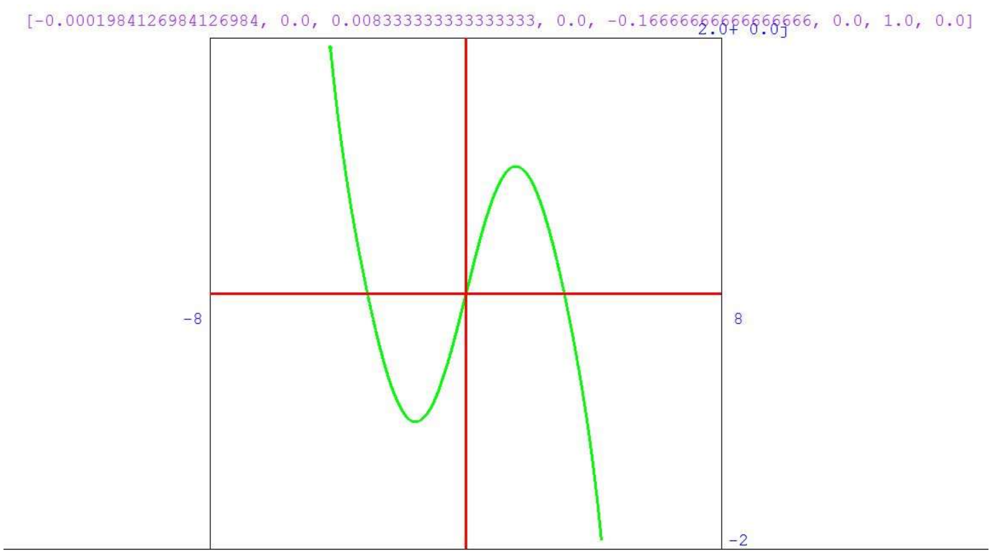
15 roots (7 real)
roots: -7.746614+3.125364j, -7.746614-3.125364j, -7.694664-7.430562j, -7.694664+7.430562j,
-7.105719, -6.416051, -3.141592, 0, 3.141592, 6.416051, 7.105719,
7.694664+7.430562j, 7.694664-7.430562j, 7.746614+3.125364j, 7.746614-3.125364j

It was not until the ninth power that pi started emerging and by the fifteen power it was good to six places. However, 2 pi was just beginning to develop. More real roots were merging, but more complex roots were developing than real routes. Even though the series that evolves does merge to pi, it does it very slowly. This makes us question this approach to use an infinity argument to derive a valid value for pi.

Now let us look more closely at the meaning of the roots of a polynomial using the following as an example:

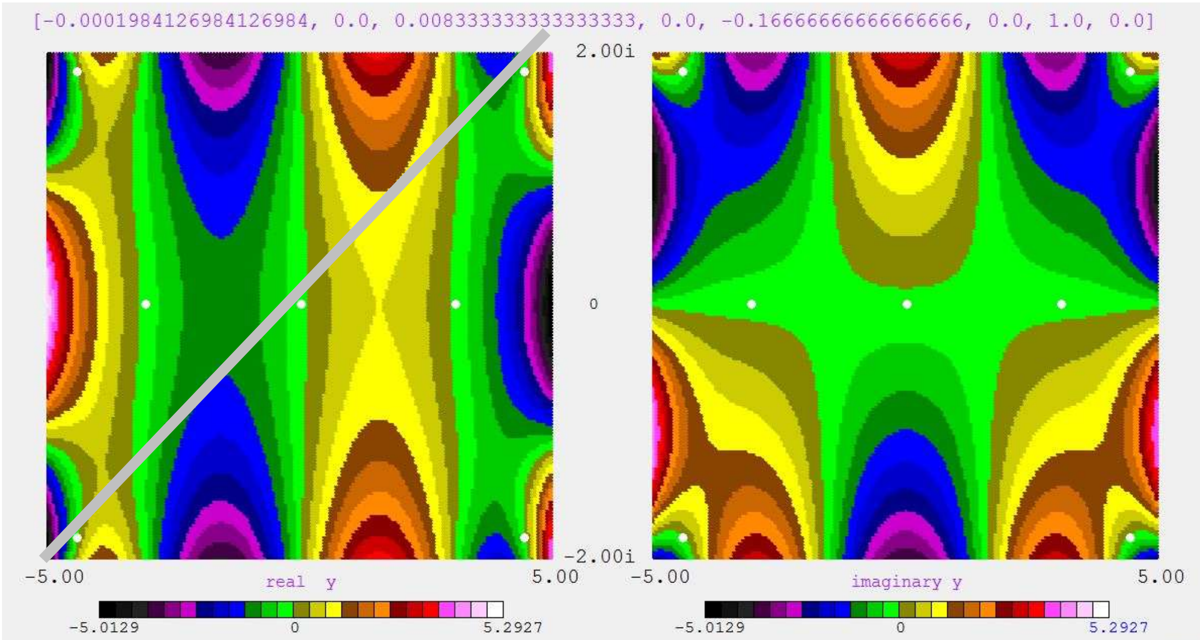
y = -x^7/7! +x^5/5!-x^3/3!+ x^/1!

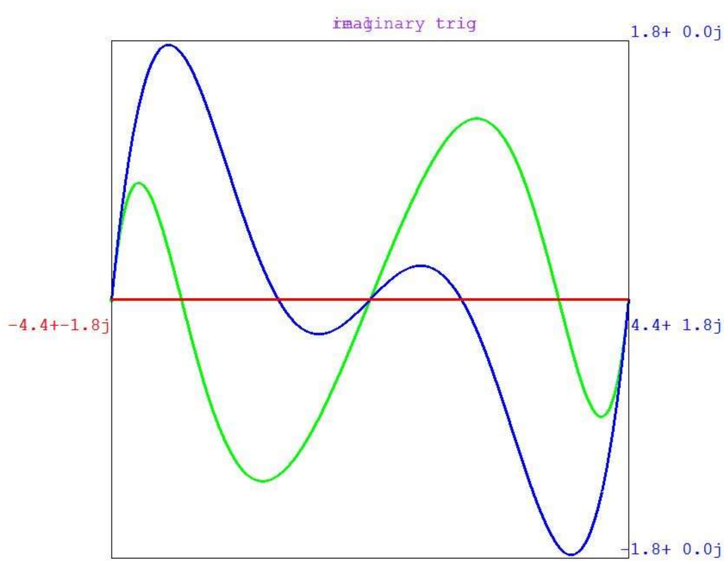
Since it is a 7th degree polynomial there should be seven answers:
roots: -4.434005+1.843752j, -4.434005-1.843752j,
-3.078642, 0, 3.078642,
4.434005+1.843752j, 4.434005-1.843752j
When I look at the curve. I only see three roots, not 7.



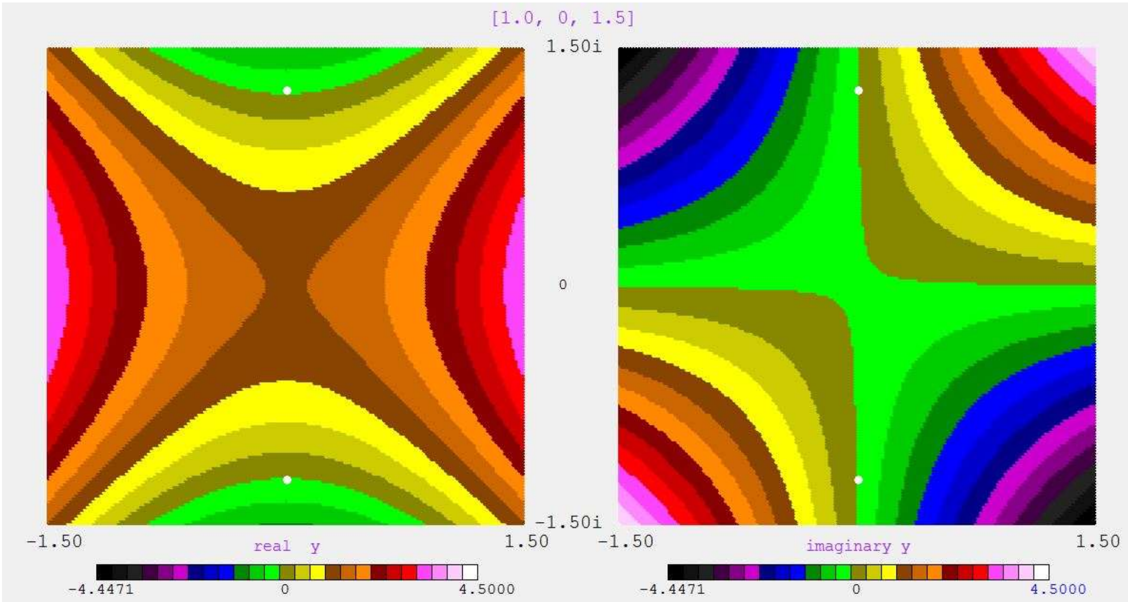
This is because we are only looking at a slice of the curve in the x-y plane. To see the rest of the crossings, we have to look in the x-xi plane. This becomes a four dimension plot in which we can only look at three of those dimensions through a contour plot. Here we are using colors to express the numeric values. The khaki green color is where the zeros are in the curve. We have drawn a gray line through two complex points (-4.434005-1.843752j , 4.434005+1.843752j) and one real point (0,0)

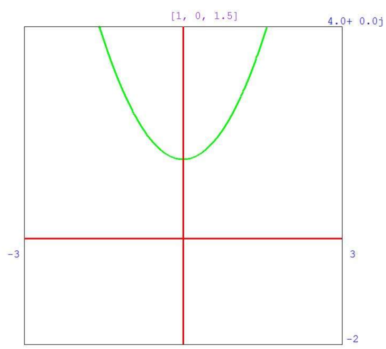
If we draw a gray line through those three points, we get the following graphs where the blue curve is the real part of y and the green curve is the imaginary part of y.

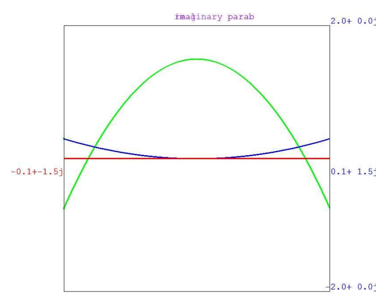


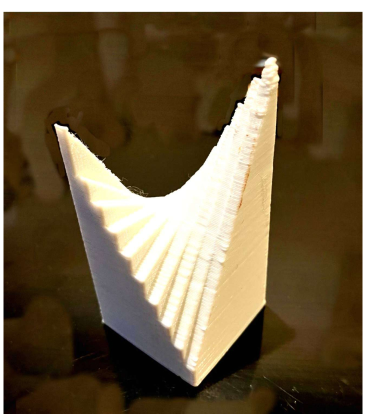


While both curves cross the zero line several times, they only cross it simultaneously, three times. Thus both the real and imaginary parts of must be zero simultaneously to be considered A root of the equation. It is because we have not used all of the presentation tools available to us that we could not comprehend what a complex zero crossing meant.   Standard plotting packages such as Demos and Mathsisfun do not handle complex numbers. We needed the contour plot or a 3d printer and the graphing program to fully understand what is happening, We begin to realize that complex numbers constitute another dimension.









Let us look at the graph for a parabola with which everyone is familiar. The top right diagram shows the contours of the real part of y. We can see a cross section of it along the real x axis. In the graph below the contours. The graph to the right of that shows y snd yi plot with a cross section through the imaginary roots. The blue line representing yi is slightly of from the zero cut so we could understand the requirement that both yi and y be zero at the same point. The figure on the bottom left is a 3D model of that contour plot.