The Twelve Labours of Minerva

1. Long Division with checking
   Explaining long multiplication
2. Proving Fraction and Negative number operations
   Showing parallel between fractions and negative numbers
3. Explain how to solve 6x6 magic square
   Show how to find over 50,000 solutions to 5x5 magic square
4. Evaluate a number raised to a decimal power **5^{th} grade**
   Find the logarithm of a number
   Derive formulas for exponentials and logarithms
5. Find the logs of numbers from 1 to 100 given log(2)=.301 and log(3)=.477
6. Find $(a + b)^n$ for n=0 to 5
   Find Pascal triangle
   Prove n!/((n-m)!m!) + n!/((n-(m+1))!)(m+1)!) = (n+1)!/((n+1-(m+1))!(m+1)!)
   Prove square root derivation
   Find e^x as a power series
7. Derive the formulas for mortgage payments **8^{th} grade**
   At what times are the hour hand and the minute hand at the same positions?
8. Prove trigonometric formula using e^x.
   Prove trigonometric formula using power series
9. Prove the derivative of ln(v)
   Derive the derivative formula
   Show integration is the inverse of differentiation
10. Derive the log tables using a power series
    Derive the cosine tables using a power series
    Write the programs in Python
11. Prove sum of angles of a triangle equal 180^0.
    Derive formula for sum of angles of a polygon.
    Prove Pythagorean theorem
12. Derive quadratic formula **12^{th} grade**
    Derive cubic formula

The Minerva challenge is opened to 3^{rd} graders to 12^{th} graders who wish to qualify as an outstanding math student. Awards will be given at local government offices on March 14 – Pi day. The answers are given below, but the student must pass an oral exam. The answers are not complete but serve as a guide.

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1. Long division

100.506	981.2	100.506	1
981.2)98616.9008	x 100.506	x 981.2	2
9812 ^	58872	201012	3
49690	49060	100506	4
49060	9812	804048	5
63008	98616.4872	904554	6
58872	+ .4136	98616.4872	7
4136	98616.9008	+ .4136	8
		98626.9008	9

In line 2 we multiplied divisor and dividend by ten to make divisor an integer.
In line 3 we seem to be subtracting 100,000 9812s, but are actually subtracting 100 981.2s , We are using an "accounting" approach
We could continue beyond line 8 by concatenating more zeros to the dividend.

In the first check, we multiplied by the quotient, because it was easier. However, we would be doing the same multiplications and possibly making the same errors.

Theory of long multiplication

123x456=123x(400+50 +6)	expanded notation
=123x(4x100+5x10+6x1)	expand notation multiplicative format
=123x(4x100)+123x(5x10)+123x(6x1)	Distributive rule
=(123x4)x100 +(123x5)x10+(123x6)x1	Assocative rule

Changing format	123	123		123	
	x 456	x456		x456	
	(123x6)x1	738x1	mult. as	738	powers of 10
	+(123x5)x10	615x10	addition	+ 615	
	+(123x4)x100	492x100		+492	
				56088	

casting out 9s 1+2+3=6 4+5+6=15=1+5=6 6x6=36=3+3=9
5+6+0+8+8=27=2+7=9

2. Prove Fraction and Negative number operations

Adding fractions

1/7+2/7+3/7= 1x1/7+2x1/7+3x1/7	notation
=(1+2+3)x1/7	distributive property
=6x1/7= 6/7	notation

Multiplying & dividing fractions

n x 1/n=1 def division

(a) 1/n=1 / n def of div if A x B = C, then B = C / A

multiplication

m= m x 1

(1) m / n = m x 1 / n =m x (1 / n) =m x 1/n

1:	n x 1/n x m x 1/m = 1 x 1 =1	definition of fractions
2:	n x m x 1/n x 1/m = 1	commutation
3:	(n x m) x (1/n x 1/m) =1	association

4: 1/n x 1/ m = 1 / (n x m) definition of division
5: 1/n x 1/m = 1/(nxm) (a)
6: p x q x 1/n x 1/m = p x q x 1/n x m multiplication
7: p x 1/n x q x 1/m = p x q x 1 / (n x m) commutation
8: p/n x q/m = (p x q)/(n x m) (1) notation

division

1: n x 1/n x m x 1/m = 1 x 1 =1 definition of fractions
2: n x 1/m x m x 1/n = 1 commutation
3: n/m x m/n =1 (1)
4: n/m = 1 / m/n definition of division

Negative numbers

n+ -n =0 def of neg. nos. definition of subtraction
(a) -n = 0 - n def of sub. if A + B =C, then B = C - A
1: n + -n + m + -m = 0 + 0 =0 defintion of negative numbers
2: n + m + -n + -m = 0 commutation
3: (n + m) + (-n + -m) = 0 association
4: -n + -m = 0 – (n+m) definition of subtraction
5: -n + -m = -(n+m) (a)

n=n+0
(2) n-m= n+ 0 –m = n+ -m

1: n + -n + m + -m = 0 + 0 =0 defintion of negative numbers
2: n + -m + m+ -n = 0 commutation
3: n-m + m-n = 0 (2)
4: (n-m) + (m-n)=0 association
5: m-n = 0 – (n-m) definition of subtraction
6: m-n = -(n-m) (1)

Parallelism

n x -n = 0
replace –n with /n or 1/n , / with x, and 0 with 1
n x /n = 1 x 1/n = 1
note - - = + and / / =x

Multiplication of negative numbers

n + -n = 0
(b) n = 0 - -n
5 x –3 = -3 + -3 + -3 + -3 + - 3 = -(3+3+3+3+3)=-(5x3)
-3 x 5 = -(3x5) computation
0 = (n – n) = n + -n
-m x 0 =-m x (n + -n) multiply
0 = -m x n + -m x –n distributive property
0 = -(m x n) + (-m x –n) association
(-m x –n)= 0 - - (n x m) definition of subtraction
-m x –n = n x m (b)

3. Explain how to solve the 6x6 magic square

Let us solve the odd number magic square

1. Start with 1 in the middle top row
2. Each move starts on a right diagonal with the next sequential number
3. If the move is within the magic square, place it on the square to which you moved
4. If move is off the board, move it to the opposite end of the row or column
5. If a number exists on the square to which you have tried to move or the square from which you moved is the upper right hand quarter square, place the number on the square directly below the number from which you started.

All the rows, columns, and major diagonals add to $n*(n^2+1)/2$, wher n = number of rows and number of columns.

	1	

rule 3

	1	
		2

rule 4

	1	
3		
		2

rule 4

	1	
3		
4		2

rule 5

	1	6
3	5	
4		2

rule 3

	1	6
3	5	7
4		2

rule 5

Now let us solve the 6 x 6 magic square.

Let us define the x pattern:

1	4
3	2

and the reverse u:

8	5
7	6

Now think of the 6x6 as 9 4x4 squares. In the left and right lower corners and the middle sets of four squares, use the reversed u pattern, and for the others use the x pattern. Follow the rule for the odd magic squares for placing the groups of four:

		1	4		
		3	2		

rule 3 and x pattern

		1	4		
		3	2		
				8	5
				7	6

rule 4 and reverse u

		1	4		
		3	2		
9	12				
11	10				
				8	5
				7	6

Rule 4 and x pattern

		1	4		
		3	2		
9	12				
11	10				
16	13			8	5
15	14			7	6

rule 5 and reverse u

		1	4		
		3	2		
9	12	20	17		
11	10	19	18		
16	13			8	5
15	14			7	6

Rule 3 and last reverse u

29	32	1	4	21	24
31	30	3	2	23	22
9	12	20	17	25	28
11	10	19	18	27	26
16	13	33	36	8	5
15	14	35	34	7	6

Final solution

All rows columns and major diagonal add to the same number: 111.

Let us go to a base 5 system:

Use diagonal approach

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Modulo 5

2	4	1	3	5
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4

Subtract 1

1	3	0	2	4
2	4	1	3	0
3	0	2	4	1
4	1	3	0	2
0	2	4	1	3

Replace 1 with 0, 3 with 1, 0 with 2 and 2 with 3

0	1	2	3	4
3	4	0	1	2
1	2	3	4	0
4	0	1	2	3
2	3	4	0	1

Shift 0 over 3 row by row

0	1	2	3	4
2	3	4	0	1
4	0	1	2	3
1	2	3	4	0
3	4	0	1	2

Use formula to get a solution: 1+ 5 * matrix 1 plus matrix 2

1	7	13	19	25
18	24	5	6	12
10	11	17	23	4
22	3	9	15	16
14	20	21	2	8

4. Evaluate a number raised to a decimal power
Find the logarithm of a number

$3.1415^{3.1415} = 36.4$

Construct a Table

1	1	3.1415
1 /2	.5	1.7724
1 /4	.25	1.3313
1/8	.125	1.1538*
1/16	.0625	1.0741
1/32	.03125	1.0364
1/64	.015625	1.0180*
1/128	.0078125	1.0089

.1415
<u>.125</u>
.0165
<u>.015625</u>
.000825

3.1415*3.1415*3.1415= 31.0035
31.0035*1.1538*1.0180=36.4

```
         1. 7 7  2
        √3.14150000
         1
  27     214
         189
  347     2515
          2429
  3542       8600
             7084
```

$\log_{10}(7) = .845$

1	1	10
1 /2	.5	3.1662*
1 /4	.25	1.7782*
1/8	.125	1.3335
1/16	.0625	1.1547*
1/32	.03125	1.0746*
1/64	.015625	1.0366
1/128	.0078125	1.0181
1/256	.00390625	1.0090

7/3.1662=2.2108
2.2108/1.7782=1.2432
1.2432/1.1547=1.0766
1.0766/1.0746=1.0018

.5+.25+.0625+.0312+.0078=.8437

5. Find the logs of numbers from 1 to 100 given log(2)=.301 and log(3)=.477

Log(5)=log(10)-log(2)=.699
Log(6)=log(2+log(3)=.778
Log(8)=3*log(2)=.903
Log(9)=2*log(3)=.954
Log(48)=log(6)+log(8)=1.681
Log(50)=log(5)+log(10)=1.699
Log(49)≈(log(50)+log(48))/2=1.690
Log(7)=log(49)/2=.845
Log(98)=log(2)+log(49)=1.991
Log(99)≈(log(100)+log(98))/2=1.995
Log(11)=log(99)-log(9)=1.041

6. Find $(a + b)^n$ for n=0 to 5
Find Pascal triangle
Prove n!/((n-m)!m!) + n!/((n-(m+1))!)(m+1)!) = (n+1)!/((n+1-(m+1))!(m+1)!)
Prove square root derivation

1	0	0	0	0	0
1	1	0	0	0	0
1	2	1	0	0	0
1	3	3	1	0	0
1	4	6	4	1	0
1	5	10	10	5	1

1
$\underline{a+b}$
$a+b$
$\underline{a+b}$
a^2+ab
$\underline{\quad ab+b^2}$
$a^2+2ab+b^2$
$\underline{\qquad a+b}$

$a^3+2a^2b+ab^2$
$\underline{\quad a^2b+2ab^2+b^3}$
$a^3+3a^2b+3ab^2+b^3$
$\underline{\qquad a+b}$
$a4+3a^3b+3a^2b^2+ab^3$
$\underline{\quad a^3b+3a^2b^2+3ab^3+b^4}$
$a^4+4a^3b+6a^2b^2+4ab^3+b^4$

n!/((n-m)!m!) +n!/(((n-(m+1))!(m+1)!)=
(n!/((n-(m+1))!(m)!)(1/(n-m) +1/(m+1))=
(n!/((n-(m+1))!(m)!)(n+1)/((n-m)(m+1))=(n+1)!/((n-m)!(m+1)!)=
(n+1)!/(((n+1)-(m+1))!(m+1)!)

$\underline{\ a \quad + \ b}$
$\sqrt{a^2+2ab+b^2}$
a2
2a+b $\quad 2ab+b^2$
$\underline{2ab+b^2}$

$$(a+b)^n=\sum_{n=0}^{\infty} n!/((n-m)!m!)\ a^{n-m}b^m \quad \text{for } 1/(-m!)=0$$

m	n!/((n-m)!(m!))
0	n!/(n!0!)=1
1	n!/((n-1)!1!)=n
2	n!/((n-2)!2!)=n(n-1)/2!
3	n!/((n-3)!3!)=n(n-1)(n-2)/2!

a=1

$(1+b)^n=1+nb+n(n-1)/2!\ b^2+n(n-1)(n-2)/3!\ b^3+\ldots$

b=d n=x/d

$(1+d)x/d=((1+d)^{1/d})^x=1+x/d\ ^*\mathbf{d + (x/d)}(x/d-1)d^2/2!\ +(x/d(x/d-1)(x/d-2)/3!+$

$=1+x+x(x-d)/2!+x(x-1)(x-2d)/3!$

Lim $d\rightarrow 0$

$$e^x=1+x+x^2/2!+x^3/3!+\ldots=\sum_{n=0}^{\infty} x^m/m!$$

7. Derive the formulas for mortgage payments

At what times are the hour hand and the minute hand at the same positions?

P1=Pr+E1

P2=P1=(P-E1)r+E2

P2-P1=0=Pr-rE1-Pr-E1+E2=-(1+r)E1+E2

E2=(1+r)E1

P3=P1=(P-E1-E2)r+E3

P3-P2=(P-E1)r-rE2-(P-E1)r-E2+E3=-(1+r)E2+E3

$E3=(1+r)E2=(1+r)^2E1$

$En=(1+r)^{\text{~~n-1~~}}E1$

$P=\Sigma(1+r)^mE1=E1(1-(1+r)^n)/((1-(1+r))=E1((1+r)^n-1)/r$

$P1=Pr+E1=rP(1+1/((1+r)^n-1))=r(1+r)^nP//((1+r)^n-1)$

d=rt $\quad 1=r_h x12 \quad 12=r_m x12$

$r_h=1/12 \quad r_m=1$

$t^*r_m=1+t^*r_h$

t*1=1+t/12

t(11/12)=1

t=12/11=1 1/11 hrs= 1 hr+60/11 min = 1 hr + (5 5/11) min

= 1hr + 5 min 300/11 sec = 1hr 5 min 27 3/11 sec

8. Prove trigonometric formula using e^x.

Prove trigonometric formula using power series

let $x=iy$ $i^0=1$ $i^1=1$ $i^2=i*i=-1$ $i^3=i^2*1=-1$ $i^4=i^2*i^2=-1*-1=1$

$e^{ix}=1+ix-x^2/2!-ix^3/3!+x^4/4!+ix5/5!-x^6/6!+.$

$=1-x^2/2!+x^4/4!-x^6/6!+..$

$i(x-x^3/3!+x5/5!-..$

$=\cos(x)+i\sin(x)$

$e^{-ix}=\cos(x)-i\sin(x)$

$e^{ix}*e^{-ix}=e^0=1=(\cos(x)+i\sin(x))*(\cos(x)-i\sin(x))=\cos^2(x)+\sin^2(x)$

$=e^{i(x+y)=}e^{ix}*e^{iy}$

$\cos(x+y)+i\sin(x+y)= (\cos(x)+i\sin(x))(\cos(y)+i\sin(y))$

$=\cos(x)\cos(y)-\sin(x)\sin(y)+i(\sin(x)\cos(y)+\sin(y)\cos(x))$

Therefore, $\cos(x+y)= \cos(x)\cos(y)-\sin(x)\sin(y)$

$\sin(x+y)= \sin(x)\cos(y)+\sin(y)\cos(x)$

$\sin(x)=x-x^3/3!+x^5/5!-$

$\underline{x\ \cos(x)=1-x^2/2!+x^4/4!-}$

$x-x^3/0!3!+x^5/0!5!-$

$-x^3/1!2!+x^5/2!3!-$ $1/0!3! +1/1!2! + 1/2!1!+ 1/3!0!= 2^3/3!$

$\underline{x^5/1!4!-}$

$x-x^3 2^2/3!+x^5 2^4/5!= 1/ 2*(2x-(2x)3/3!+(2x)5/5!)=1/2 \sin(2x)$

$\sin(2x)=2\sin(x)\cos(x)$

9. Prove the derivative of ln(v)

Derive the derivative formula

Show integration is the inverse of differentiation

$y=\ln(u)$

$y+\Delta y=\ln(u+\Delta u)$

$\Delta y=\ln(u+\Delta u)-\ln(u)=\ln(1+\Delta u/u)$

$=\Delta u/u * u/\Delta u \ln(1+\Delta u/u)= \Delta u/u \ln(1+\Delta u/u)^{u/\Delta u}$

$\Delta y/\Delta x= 1/u \ln(1+\Delta u/u)^{u/\Delta u}\ \Delta u/\Delta x$

$\lim \Delta x\rightarrow 0$

$dy/dx= 1/u \ln(e)\ du/dx = 1/u\ du/dx$

$y=v * u$

$\ln(y)=\ln(v * u)= \ln(v)+\ln(u)$

$1/y\ dy/dx = 1/v\ dv/dx + 1/u\ du/dx$

$dy/dx = y/v\ dv/dx + y/u\ du/dx$

$=u\ dv/dx + v\ du/dx$

$y =v^u$

$\ln y = u \ln(v)$

$1/y\ dy/dx = \ln(v)\ du/dx + u/v\ dv/dx$

$dy/dx = y * \ln(v)\ du/dx + y* u/v\ dv/dx$

$= \ln(v)\ v^u\ du/dx + u * v^{u-1}\ dv/dx$

$\sin(x) = (e^{ix}-e^{-ix})/2i$

$d \sin(x)/dx = i(e^{ix}+e^{-ix})/2i=(e^{ix}+e^{-ix})/2=\cos(x)$

$\Delta f(x)/\Delta x\ \ \Delta x\ = \Delta f(x) = f(x) - f(x_0)$

10. Derive the log tables using a power series
    Derive the cosine tables using a power series
    Write the programs in Python

```
def myln(xi):
    e=mye()
    inv=0
    x=xi
    if x<1.:
       inv=1
       x=1./x
    root=e
    i=0
    while i<5:
        root=mysqrt(root)
        i=i+1
    i=0.
    while x>e:
        x=x/e
        i=i+1.
    j=0.
    while x>root:
        x=x/root
        j=j+1.
    x=man(x)
    x=x+i+j/32.
    if inv==1:
       x=-x
    return x
```

```
def man(xi):
    x=xi
    x=(x-1.)/(x+1.)
    x2=x*x
    k=11.
    c=x2/k
    k=k-2.
    while k>1.:
        c=(1./k+c)*x2
        k=k-2.
    c=2*(1.+c)*x
    return c
def mysqrt(numi):
    inv=1
    pt=0
    x=0
    x2=numi
    if x2!=0.:
        if x2>10.:
            x2=1./x2
            inv=-1
     while x2<1.:
            x2=100.*x2
            pt=pt+1
        x=1.+(x2-1.)/2.
        k=0
        while k<7:
            x=.5*(x+x2/x)
            k=k+1
        k=pt
        while k>0:
         x=x/10.
            k=k-1
        if inv==-1:
            x=1./x
    return x

def mye():
    e=.1
    x=30.
    while x>0.:
        e=1.+(1./x)*e
        x=x-1.
    return e
```

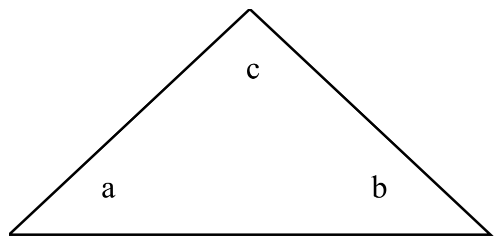
```
def mycos(i):
    pie=mypi()
    if(i>45.):
        x=(90.-i)*pie/180.
        x2=x*x
        c=1.
        j=16.
        while j>0.:
            c=1.-(x2/(j*(j+1.)))*c
            j=j-2.
        c=x*c
    else:
        x=i*pie/180.
        x2=x*x
        c=1.
        j=14.
        while j>0.:
            c=1.-(x2/(j*(j-1.)))*c
            j=j-2.
    return c

def mypi():
    i=2
    c=mysqrt(2.)
    while i>0:
        x=2.-c
        s=mysqrt(x)
        x=2.+c
        c=mysqrt(x)
        i=i-1
    x=s/c
    x2=x*x
    k=21.
    hold=1./k
    k=k-2.
    while k>0:
        hold=1./k-x2*hold
        k=k-2.
    hold=(x*hold)*16.
    return hold
```

11. Prove sum of angles of a triangle equal 180^0.

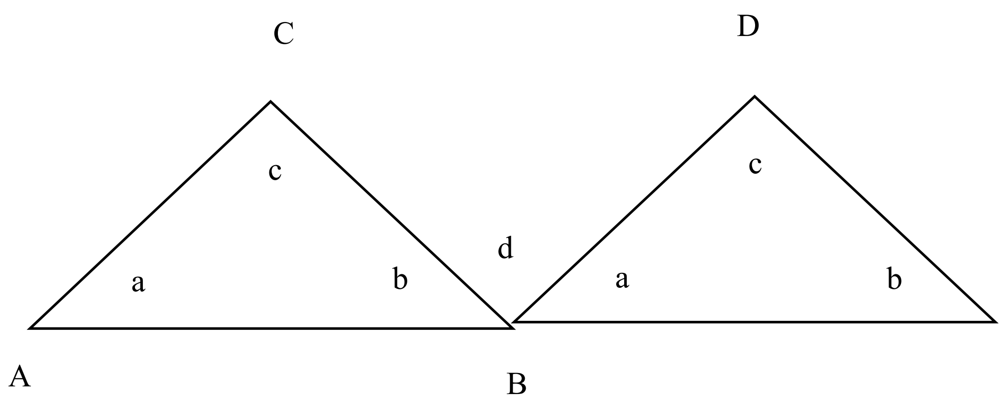
Derive formula for sum of angles of a polygon.

Prove Pythagorean Theorem

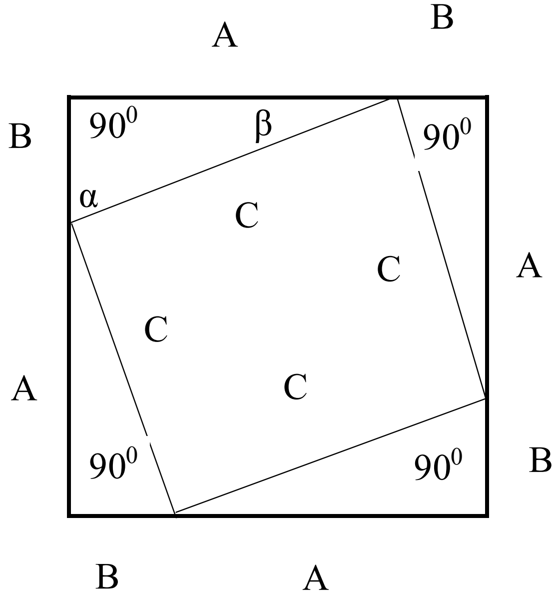
The sum of the angles of a triangle are equal to the measure of a straight line.



Slide the triangle on straight line so that the vertex of angle a touches that of angle b.



Line AC is parallel to line BC	They make the same a with 3rd line
<c = <d	Alternate interior angle of || lines
<a+ <b+ <d = 180^0	Whole = sum of parts
<a+ <b+ <c = 180^0	Substituting an equal



$\alpha+\beta=90^0$	sum of angles of triangle = 180 degrees
The angles of the polygon=90^0	angle of a line = 180^0
The polygon is a square	sides are all equal and angles are all right angles
$(A+B)^2=C^2=4*(1/2\ AB)$	whole is equal to sum of parts
$A^2+2AB+B^2=C^2+2AB$	Expanding operations
$A^2+B^2=C^2$	Subtract 2AB from both sides

12. Derive quadratic formula
    Derive cubic formula

$ax^2+bx+c=0$

$x^2+(b/a)x+c/a=0$

$x^2+(b/a)x+\mathbf{(b/2a)^2-(b/2a)^2}+c/a=0$

$(x+b/2a)^2-(b/2a)^2+c/a=0$

$(x+b/2a)^2 = (b/2a)^2-c/a = (b/2a)^2-4ac/(4a^2) = (b^2-4ac)/(4a^2)$

$x+b/2a = \pm(b^2-4ac)^{1/2}/2a$

$\mathbf{x \quad =(b\pm(b^2-4ac)^{1/2})/2a}$

$\mathbf{x^3-10x^2+31x-30=0}$

$x=y-(-10/3)=y+10/3$

$\mathbf{y^3-7/3y-20/27=0}$ $\qquad \sin^3(\theta)-3/4\sin(\theta)+1/4\sin(3\theta)$

$y=az$

$a^3z^3-7/3az-20/7=0$

$z^3-((7/3)/a^2)z-20/(27a^3)=0 \qquad -7/(3a^2)=-3/4$

$a=2/3\ (7^{1/2}) \qquad (-5/2)/(7^{3/2})=1/4\sin(3\theta)$

$\sin(3\theta)=-10/\ 7^{3/2}$

$\sin(3\theta)=(-10)/7^{3/2}$
$\theta=1/3\ \sin^{-1}((-10)/7^{3/2})$
$\theta=1/3\ i\ \ln(iu+(1-u^2)^{1/2})\quad u=(-10)/7^{3/2}$
$\theta=i\ln(((-10)/7^{3/2}\ i+(1-100/7^3)^{1/2})^{1/3})$
$\quad=i\ln((-10i+9(3^{1/2})^{1/3}/7^{1/2})$

$\sin(3\theta)=(-10)/7^{3/2}=-0.539949247$

n=0 n=1 n=2

$3\theta+2n\pi=-0.57037681,\ 5.7128085,\ 11.9959938$
$\theta=\ -0.19012560,\ 1.9042695\ ,\ 3.9986646$
$\sin(\theta)=-0.188982233,\ 0.944911183,\ -0.755928946$
$y=az\quad a=2/3\ (7^{1/2})=1.7638342073$
y=-0.3333333, 1.66666667, -1.33333333
$x=y+10/3=y+3.3333333$
x=3,5,2

$\theta=-i\ln((-10i+9(3^{1/2}))^{1/3}/7^{1/2})$
$1/(iu+(1-u^2)^{1/2})=(iu-(1-u^2)^{1/2})/(-u^2-(1-u^2))$
$\quad=(1-u^2)^{1/2}-iu$
$\sin(\theta)=(e^{i\theta}-e^{-i\theta})/2i$
$\quad=((-10i+9(3^{1/2}))^{1/3}-(9(3^{1/2})+10i)^{1/3}))/(2(7^{1/2})i)$
$\quad$**$(-(3^{1/2}+2i)1^{1/3}+(3^{1/2}-2i)1^{-1/3}$ **$)/(2(7^{1/2})i)$
$1^{1/3}=1:\qquad z=\sin(\theta)=-4/(2*7^{1/2})$
care with complex form of $1^{1/3}$ and $1^{-1/3}$
$1^{1/3}=(-1+3^{1/2}i)/2:\quad z=\sin(\theta+120^0)=-1/(2*7^{1/2})$
$1^{1/3}=(-1-3^{1/2}i)/2:\quad z=\sin(\theta+240^0)=5/(2*7^{1/2})$

$1^{1/3}=(-1+3^{1/2}i)/2:\quad 1^{-1/3}=2(1+3^{1/2}i)/((1+3^{1/2}i)(-1+3^{1/2}i))=-(1+3^{1/2}i)/2$
$1^{1/3}=(-1-3^{1/2}i)/2:\quad 1^{-1/3}=2(1-3^{1/2}i)/(1-3^{1/2}i)(-1-3^{1/2}i)=(-1+3^{1/2}i)/2$

$-(3^{1/2}+2i)1^{1/3}+(3^{1/2}-2i)1^{-1/3}=$**$-4i$**
$(-(3^{1/2}+2i)(-1+3^{1/2}i)+(3^{1/2}-2i)(-1-3^{1/2}i))/2=(-i+33^{1/2}+-i-33^{1/2})/2=$**$-i$**
$(-(3^{1/2}+2i)(-1-3^{1/2}i)+(3^{1/2}-2i)(-1+3^{1/2}i))/2=(5i-3^{1/2}+5i+3^{1/2})/2=$**$5i$**

$x=y+10/3$
$y=az\quad a=2/3\ (7^{1/2})$
$z=\ -4/(2*7^{1/2}),\quad -1/(2*7^{1/2})\quad,\quad 5/2*(7^{1/2})$
$y=-4/3\quad,\quad -1/3\quad,\quad 5/3$
$x=2\quad,\quad 3\quad,\quad 5$