The Identity Elements

Zero-the additive identity

Zero has the unity property in addition because it is the integer before 1 and 1 is used to create the integers.

Let ? be the number before 1.

1+1 =2
1+1+1 =3=(1+1)+1 =2+1
=1+(1+1) =1+2
1+1+1+1=4=(1+1+1)+1=3+1
+1+(1+1+1)=1+3

We see that we can create the next number by adding 1 to the left or right of the number (commutation) to create the next number. Therefore, ?+1=1 or 1+?=1 making ? commutative.

4=1+3=?+1+3=?+(1+3)=?+4 or
4=3+1=3+1+?=(3+1)+?=4+?

Thus a number added to ?, the number before 1 gives the number to which ? was added. We have learned to call ?, zero (0).

Zero added to itself

We had to make use of the commutative, associative, and transitive properties of numbers to prove that zero was an identity element.

1+1=2=2+0
1+0+1+0=1+1+0+0=2+(0+0)
By the transitive property 0=0+0

One-the multiplicative identity

One has the unity property in multiplication because of how we define multiplication. Multiplication is define as repeated addition:

One

1x5= 5=0+5 identity
2x5=5+5=0+5+5

5x1=0+1+1+1+1+1=0+5=1x5 identity
5x2=2+2+2+2+2=(1+1)+ (1+1)+ (1+1)+ (1+1)+ (1+1)=(1+1+1+1+1)+(1+1+1+1+1)
=5+5=2x5
We can see that multiplication is commutative.

Zero

5x0=0+0+0+0+0=0
Since multiplication is commutative, then 0x5=0.
Let us look at this in a different way.
2x5=0+5+5
1x5=0+5
0x5=0 by pattern recognition
The proof is not as strong for the zero as for proof that one was the identity element for addition.

Looking at zero in other uses

Factorials

1!=1
2!=2x1=2x1!
3!=3x2x1==3x(2x1)=3x2!
4!=4x3x2x1=4x(3x2x1)=4x3!

By pattern
1!=1x0! , but
1!=1

There for 0!=1 since 1x1=1

Exponents

2^4=2x2x2x2
2^3=2x2x2 =2^4/2
2^2=2x2 =2^3/2
2^1=2 =2^2/2
2^0 =2^1/2=2/2=1
2^{-1}= =2^0/2=1/2
2^{-2}= =2^{-1}/2=1/4
We again have used pattern recognition to show that 0! And any number raised to the zero power gives 1 as the answer.

Division

$1x0=0$ $1=0/0$ division is inverse multiplication
$2x0=0$ $2=0/0$
$3x0=0$ $3=0/0$

$0x0=0$ $0= 0/0$
Division of zero by itself is indeterminant.

$0/1=0/2=0/3=0$ Zero divided by any integer is zero.

For the time being, zero divided into any number other than itself is undefined.

$0^3=0x0x0=0$
$0^2=0x0=0$
$0^1=0$

$0^{-3}=1/0^3$ undefined
$0^{-2}=1/0^2$
$0^{-1}=1/0^1$

$0^0=1/0$ coming from positive numbers or $0^1/0=0/0$ coming from negative numbers.
Thus 0/0 is undefined or indeterminant.