Long Multiplication

I. Using addition for single digit multiplication

Long multiplication combines our knowledge of addition and the multiplication facts. As a result, the problems can serve as examples to help us remember these basic facts. We begin by utilizing the definition of multiplication—repeated addition.

213	we notice that 3 3s is 3x3=9
213	we notice that 3 1s is 3x1=3, and
+213	we notice that 3 2s is 3x2=3
639	

As a multiplication we could write this as:

213
x 3
639

and do the same operations as we did in the addition example.

Now let us extend this to carry digits:

As Addition	As multiplication	As multiplication without regrouping digits
1 2	1 2	
235	235	235
235	x 5	x 5
235	1175	1175
235		
+235		
1175		

The first example is only for illustration and not for practice. Then practice the second example until you get a feel for the layout of the problem. The third example is the way we finally want you to do it where you keep the regrouping digits in your head. With the comparison to long addition, it now makes it easier to understand what you are doing.

II. Using the associative property for multiplying by powers of 10

Now let us multiply by a two digit number that is a multiple of 10.

first step	Second step	Second example
235	235	235
x 50	x 50	x500
0	11750	117500

When we multiply by ten, we just concatenate a zero to the end of the number. Thus, in the first step we place a zero under the zero. If there were more zeros, we would continue doing this until we came to the first now zero number. Note that 50 is 5 x10.

Thus 235 x 50 = 235 x 5 x 10 = (235 x 5) x 10 . We are using the associative property of numbers to explain why we can do this.

III. Using expanded notation and the distributive property

Note that when we multiplied by the five we placed the result of the first multiplication (5x5) under the five of the multiplier. In the second example we use 500 as the multiplier so that you can see what we do with several zeros.
Let us now look at another example:

first five	second five	third five	Add the three products
235	235	235	235
x55**5**	x5**5**5	x**5**55	x555
1175	1175	1175	1175
	117**5**0	11750	11750
		117**5**00	117500
			130425

We have used the distributive property 555=500+50+5 (when using expanded notation) and multiplied 235 times each one of them. It is neater to omit the trailing zeros in the second and third line of the products but not in the first line. Look at the two examples:

235	235
x555	x550
1175	11750
1175	1175
1175	129250
130425	

It is important to remember that we always start the multiplication under the number by which we are multiplying since we are not using the zeros to align the numbers.

If the zero is not at the end in the multiplier, let us see what we do.

235	235
x5005	5005
1175	1175
000	1175
000	1176175
1175	
1176175	

When the zeros are in the middle of the number, we van ignore them as illustrate by the example. We finish by doing a multiplication with all the variations

1234567
x7007500
6172835

8641996
864196

872844168 835

After we multiply by the first seven, we can copy the results for the second one. Now you practice this problem over and over while concentrating on the format or layout. We are not covering how to find errors, but it is very important to check tour work.

IV. The theory

While understanding the theory behind the multiplication does not help us do a better job, it does help us if we want to discover or learn beyond what we are taught. Let us summarized what we have done.

Multiplication as addition	235+235+235=3x235
Associative/commutative property	235x500=235x5x100=(235x5)x100=1175x100
Powers of 10	1175x100=117500
Expanded notation	235x123=235x(100+20+3)
Distributive property	235x(100+20+3)=235x100+235x20+235x3

Do this over and over until you can do if from memory. Remember mapped with one “p“. This process of using a pattern to remember it, and then practicing will help us understand once we can describe what we have learned to another person.