Deriving the Quadratic Formula

We do this proof as we would do a proof in geometry where we write the step on the left and explain it on the right.

$$ax^2+bx+c=0$$

$x^2+b/a\ x+c/a=0$	**Divide by a making coefficient of x be 1.**
$x^2+b/a\ x+(b/2a)^2-(b/2a)^2+ca=0$	**Add $0=(b/2a)^2-(b/2a)^2$**
$(x+b/2a)^2-(b/2a)^2+c/a=0$	**Complete the square**
$(x+b/2a)^2=(b/2a)^2-c/a$	**Subtract $-(b/2a)2+c/a$ from both sides of =**
$(x+b/2a)^2=(b/2a)^2-4a/4a * c/a$	**Multiply by $1=4a/4a$**
$(x+b/2a)^2=(b/2a)^2-4ac/4a^2$	**Multiply the fractions**
$x+b/2a=((b/2a)^2-4ac/4a^{2)1/2}$	**Take square root of both sides**
$x=-b\pm((b/2a)^2-4ac/4a^{2)1/2}$	**Subtract b/2a from both sides of =**
$x=(-b\pm(b^2-4ac)^{1/2})/2a$	**Factor out $4a^2$ from radical**

The two roots are: $x=(-b\pm(b^2-4ac)^{1/2})/2a$

Let us try some examples:

1) $2x^2-8x+6=0$
2) $3x^2-12x+12=0$
3) $x^2-10x+34=0$

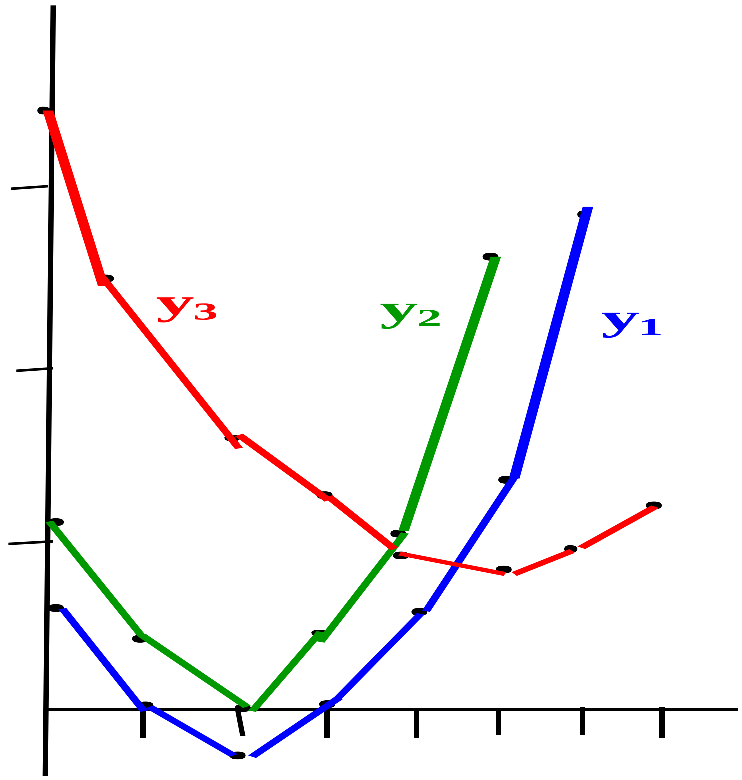
Solutions

1) $x=(8\pm(64-48)^{1/2})/4=(8\pm4(4-3)^{1/2})/4=1, 3$
2) $x=(12\pm(144-144)^{1/2})/6=2$
3) $x=(10\pm(10^2-4x34)^{1/2})/2=5\pm(25-34)^{1/2}=5\pm3i$

Table

x	y_1	y_2	y_3
0	6	12	34
1	0	3	25
2	-2	0	18
3	0	3	13
4	6	12	10
5	16	27	9
6	30	48	10
7	48	75	13

You will note that function 1 has two crossing, function 2, 1 crossing, and function 3 no crossings of the x-axis. You will also notice that the smaller is the coefficient of the x^2, the wider is the curve. The low point on the curve is $x=-b/2a$, and $y=(4ac-b^2)/4a$.


y3
y2
y1